2 Networks and paths

Syllabus topic — N1.1 Networks, N1.2 Shortest paths

This topic will develop your skills to be able to identify and use network terminology and to solve problems involving networks.

Outcomes

- Identify and use network terminology.
- Recognise the circumstances when networks can be used to solve a problem.
- Draw a network to represent a map.
- Draw a network to represent information given in a table.
- Define a tree and a minimum spanning tree for a given network.
- Determine and use minimum spanning trees to solve problems.
- Identify the shortest path on a network diagram.
- Recognise when the shortest path is not necessarily the best path.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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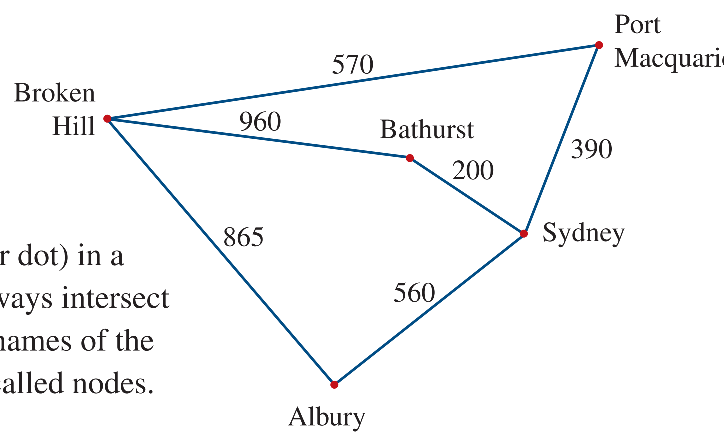
2A Networks

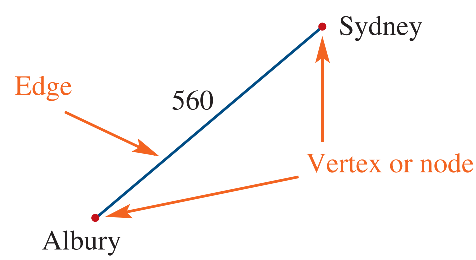
A network is a term to describe a group or system of interconnected objects. There are many situations in everyday life that involve connections between objects. Cities are connected by roads, computers are connected to the internet and people connect to each other through being friends on social media. The diagram below shows the distances in kilometres between some NSW cities. This is referred to as a network or a network diagram. Note that the lengths of the lines in a network diagram are not generally drawn to scale.

Networks: Basic concepts Watch the video in the Interactive Textbook for an illustration of the terms and concepts in action.

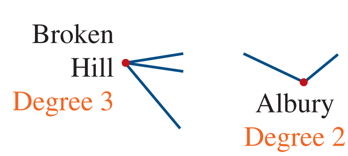
The common terms used in a network are described below.



- A **network diagram** is a representation of a group of objects called vertices that are connected together by lines. Network diagrams are also called **graphs**.
- A **vertex (plural: vertices)** is a point (or dot) in a network diagram at which lines of pathways intersect or branch. In the diagram opposite, the names of the cities are the vertices. Vertices are also called nodes.
- An **edge** is the line that connects the vertices. In the diagram opposite, the line marked with 560 is an edge. Edges can cross each other without intersecting at a node.



- The **degree** of a vertex is the number of edges that are connected to it. The degree of the Broken Hill vertex is 3 because there are three edges attached to the vertex. This is written as deg(Broken Hill) = 3.
  The degree of a vertex is either even or odd.
  - The degree of a vertex is even if it has an even number of edges attached to the vertex. For example, in the above network diagram the vertices of even degree are Port Macquarie, Bathurst and Albury.



  - The degree of a vertex is odd if it has an odd number of edges attached to the vertex. For example, in the above network diagram the vertices of odd degree are Sydney and Broken Hill.
  - There can be multiple edges between vertices, as shown.

- A **loop** starts and ends at the same vertex as shown in the diagram. It counts as one edge, but it contributes two to the degree of the vertex.

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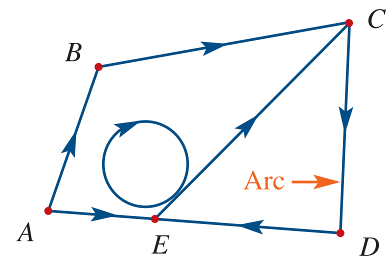
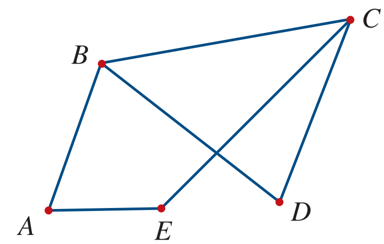
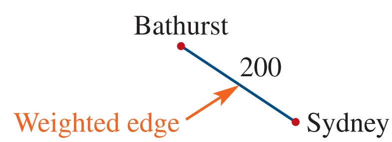
- A **directed** edge, also called an arc, has an arrow and travel is only possible in the direction of the arrow. An **undirected** edge has no arrow and travel is possible in both directions. A network or graph may have both directed and undirected edges.



- In a **directed network** or graph all the edges are directed, as in the diagram opposite, which has five vertices and six edges (arcs). It shows a path can be taken from *A* to *B* to *C*, however there is no path from *C* to *B* to *A*.
- In an **undirected network** or graph all the edges are undirected and travel on an edge is possible in both directions. The diagram opposite is an undirected graph with five vertices and six edges. It shows that there is a path from *A* to *B* and from *B* to *A*.



- In a **simple network** like the one opposite there are no multiple edges or loops.
- **Labelling of vertices:** in addition to labelling vertices on a diagram, 'labelling of vertices' in a network means listing them all in curly brackets like this, using the network above as an example: $V = \{A, B, C, D, E\}$.
- **Labelling of edges:** An edge between vertex *A* and *B* would be labelled (A, B). A loop at *B* would be (B, B). A complete list of edges for the diagram above would be $E = (A, B), (B, C), (B, D), (C, D), (C, E), (A, E)$.
- A **weighted edge** is an edge of a network diagram that has a number assigned to it that implies some numerical value such as cost, distance or time. The diagram opposite shows a weighted edge that indicates a distance of 200 km between Sydney and Bathurst. See also the first network on the previous page.



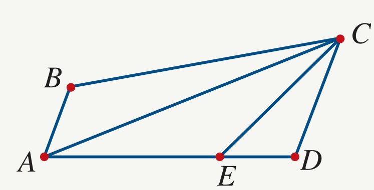
NETWORK

A network is a term to describe a group or system of interconnected objects. It consists of vertices and edges. The edges indicate a path or route between two vertices.

Example 1: Identifying and using network terminology 2A

For the network shown opposite, find the:

a number of vertices
b number of edges
c degree of vertex *C*
d number of vertices of odd degree.



SOLUTION:

1	Count the dots in the network diagram.	**a**	Five vertices
2	Count the lines in the network diagram.	**b**	Seven edges
3	Count the number of edges connected to *C*.	**c**	$\deg(C) = 4$
4	Count the number of edges for each vertex.	**d**	*A* 3, *B* 2, *C* 4, *D* 2, *E* 3
5	List the vertices of odd degree.		Two vertices of odd degree (A, E)

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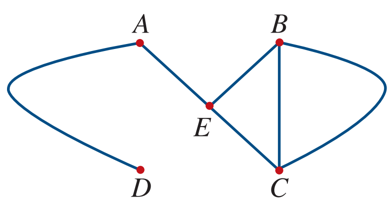
Exercise 2A

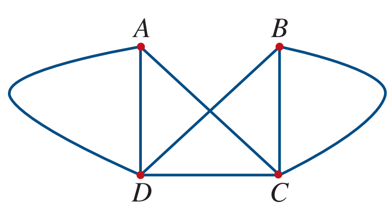
1 Copy and complete the following sentences:
a A network is a term to describe a group or system of ___________ objects.
b In a network diagram the vertices are connected together by lines called ___________.
c A ___________ is a point in a network diagram at which lines of pathways intersect.
d The ___________ of a vertex is the number of edges that are connected to it.
e A directed graph is when the edges of a network have ___________.

2 True or false?
a Vertices are represented as a point or dot in a network diagram.
b Directed networks are a connected sequence of the edges showing a route between vertices.
c A loop starts and ends at the same vertex.
d The degree of a vertex is either even or odd.
e Degree of a vertex is odd if it has an odd number of vertices attached to the edges.
f If an edge has a number assigned to it is called a directed edge.
g The edges in a directed network are usually called arcs.

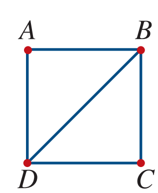
Example 1
3 Using the network diagram shown find the:
a number of vertices
b number of edges
c degree of vertex A
d degree of vertex B
e degree of vertex C
f degree of vertex D
g degree of vertex E.



4 Using the network diagram shown find the:
a number of vertices
b number of edges
c degree of vertex A
d degree of vertex B
e degree of vertex C
f degree of vertex D.



5 Using the network diagram shown find:
a $\deg(A)$
b $\deg(B)$
c $\deg(C)$
d $\deg(D)$
e the sum of the degrees of all the vertices
f the number of edges.



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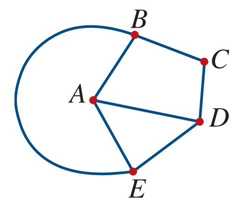
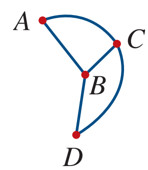
6 Using the network diagram shown find:

a deg(A)
b deg(B)
c deg(C)
d deg(D)
e the sum of the degrees of all the vertices
f the number of edges.



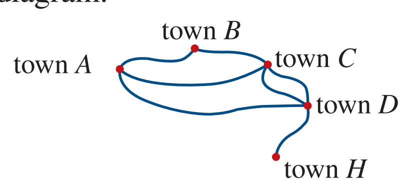
7 Using the network diagram shown find:

a deg(A)
b deg(B)
c deg(C)
d deg(D)
e the sum of the degrees of all the vertices
f the number of edges.



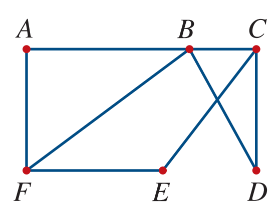
8 Find the degree of the following towns in the network diagram.

a A
b B
c C
d D



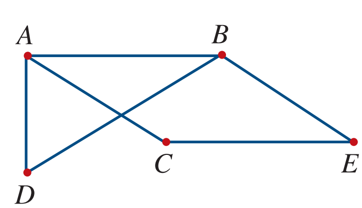
9 Using the network diagram shown find the:

a vertex with the largest degree
b vertex with the smallest degree
c vertices with an even degree
d vertices with an odd degree.



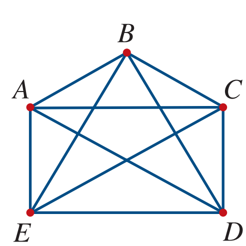
10 Using the network diagram shown find the:

a vertex with the largest degree
b vertex with the smallest degree
c vertices with an even degree
d vertices with an odd degree.



11 Using the network diagram shown find the:

a vertex with the largest degree
b vertex with the smallest degree
c vertices with an even degree
d vertices with an odd degree.



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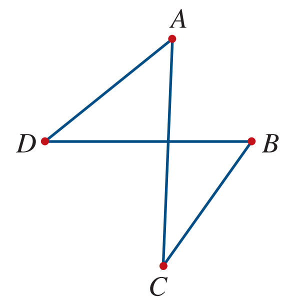
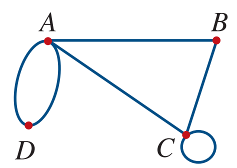
12 Using the network diagram opposite, find the:

- **a** number of vertices
- **b** number of edges
- **c** degree of vertex A
- **d** degree of vertex B
- **e** degree of vertex C
- **f** degree of vertex D
- **g** number of vertices of odd degree
- **h** number of vertices of even degree.



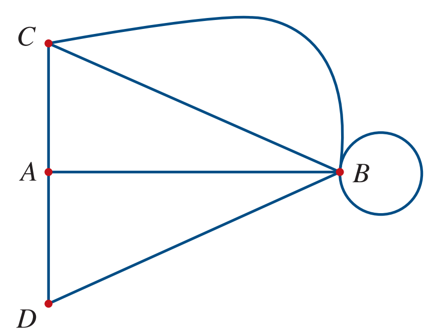
13 Using the network diagram opposite, find the:

- **a** number of vertices
- **b** number of edges
- **c** degree of vertex A
- **d** degree of vertex B
- **e** number of vertices of odd degree
- **f** number of vertices of even degree.



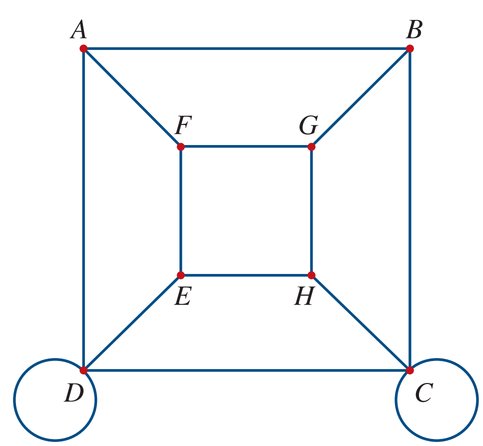
14 Using the network diagram opposite, find the:

- **a** number of vertices
- **b** number of edges
- **c** degree of vertex B
- **d** degree of vertex D
- **e** number of vertices of odd degree
- **f** number of vertices of even degree.



15 Using the network diagram opposite, find the:

- **a** number of vertices
- **b** number of edges
- **c** degree of vertex A
- **d** degree of vertex C
- **e** degree of vertex F
- **f** number of loops
- **g** number of vertices of odd degree
- **h** number of vertices of even degree.



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2B Travelling a network

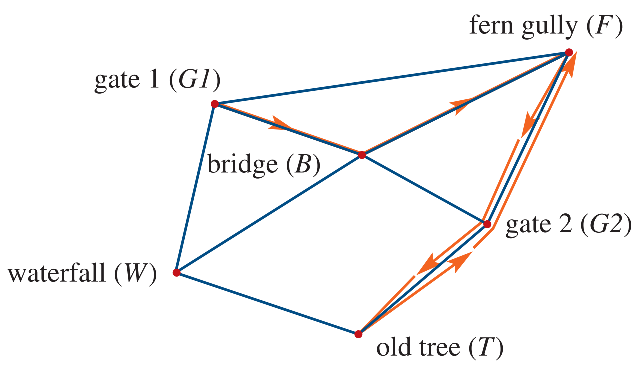
Many practical problems, such as travel routes, that can be modelled by a network involve moving around a graph. To solve such problems you will need to know about a number of concepts to describe the different ways to travel a network.

- A **walk** is a connected sequence of the edges showing a route between vertices where the edges and vertices may be visited multiple times. When there is no ambiguity, a walk in a network diagram can be specified by listing the vertices visited on the walk.

  For example, the network diagram opposite shows a walk in a forest. The forest tracks are the edges (shown in blue) and the places in the forest are the vertices. The red arrows trace out a walk in the forest and is stated as:

  $G1\text{-}B\text{-}F\text{-}G2\text{-}T\text{-}G2\text{-}F$

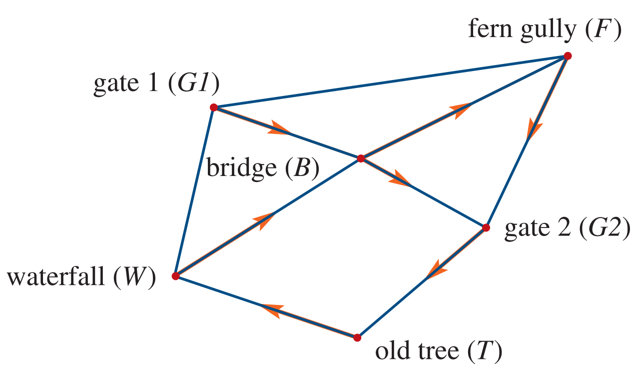
  Note: A walk does not require all of its edges or vertices to be different.



- A **trail** is a walk with no repeated edges. For example, the network diagram opposite shows a trail in a forest. The red arrows trace out a trail in the forest and is stated as:

  $G1\text{-}B\text{-}F\text{-}G2\text{-}T\text{-}W\text{-}B\text{-}G2$

  Note: A trail has no repeated edges, however there are two repeated vertices (B and $G2$).

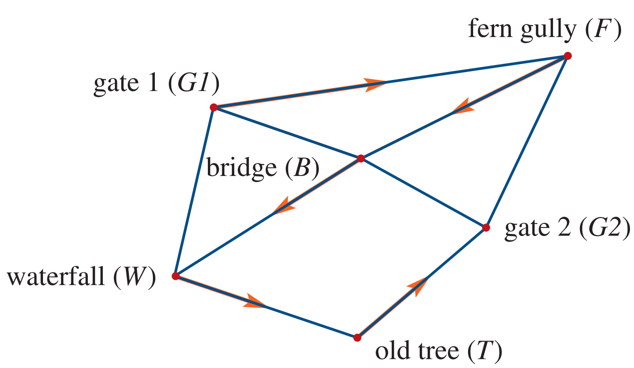


- A **path** is a walk with no repeated vertices. Open paths start and finish at different vertices while closed paths start and finish at the same vertex. Closed paths are also called circuits.

  For example, the network diagram opposite shows a path in a forest. The red arrows trace out a path in the forest and is stated as:

  $G1\text{-}F\text{-}B\text{-}W\text{-}T\text{-}G2$

  Note: A path has no repeated edges or vertices.



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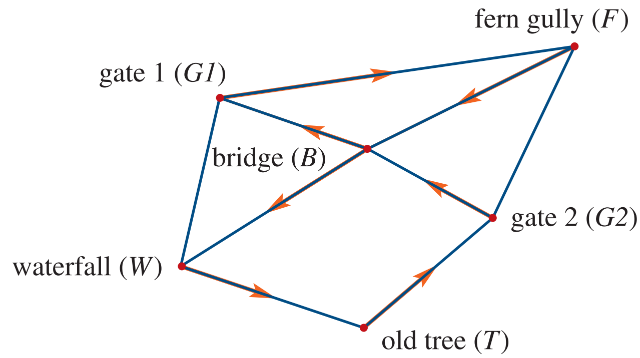
- A **circuit** is a walk with no repeated edges that starts and ends at the same vertex. Circuits are also called closed trails. Alternatively, open trails start and finish at different vertices.

  For example, the network diagram opposite shows a circuit in a forest. The red arrows trace out a circuit in the forest and is stated as:

  $$G1\text{-}F\text{-}B\text{-}W\text{-}T\text{-}G2\text{-}B\text{-}G1$$

  Note: This circuit starts and ends at the same vertex $(G1)$. There are no repeated edges however the circuit passes through the vertex B twice.

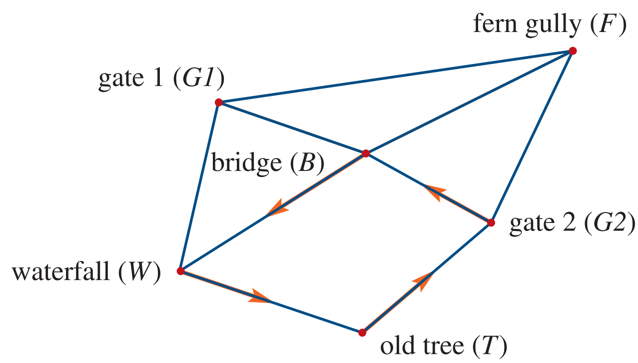


- A **cycle** is a walk with no repeated vertices that starts and ends at the same vertex. There are no repeated edges in a cycle as there are no repeated vertices. Cycles are closed paths.

  For example, the network diagram opposite shows a cycle in a forest. The red arrows trace out a cycle in the forest and is stated as:

  $$G2\text{-}B\text{-}W\text{-}T\text{-}G2$$

  Note: This cycle starts and ends at the same vertex $(G2)$. There are no repeated vertices or edges.



Travelling through a network Watch the video in the Interactive Textbook to see the five types of routes that can be travelled through networks.

TRAVELLING A NETWORK

Walk is a connected sequence of the edges showing a route between vertices and edges.
Trail is a walk with no repeated edges.
Path is a walk with no repeated vertices.
Circuit is a walk with no repeated edges that starts and ends at the same vertex.
Cycle is a walk with no repeated vertices that starts and ends at the same vertex.

Type of route	Are repeated edges permitted?	Are repeated vertices permitted?
Walk	Yes	Yes
Trail	No	Yes
Path	No	No
Circuit	No	Yes
Cycle	No	No (except first and last)

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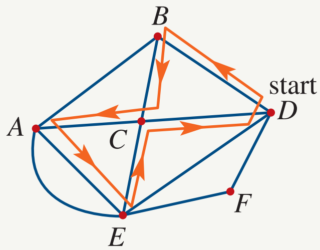
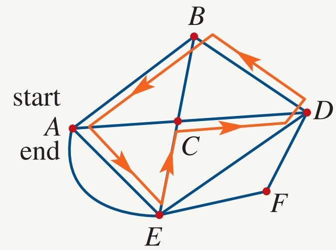
Example 2: Identifying walks, trails, paths, circuits and cycles 2B

Identify the walk in each of the graphs below as a trail, path, circuit, cycle or walk only.

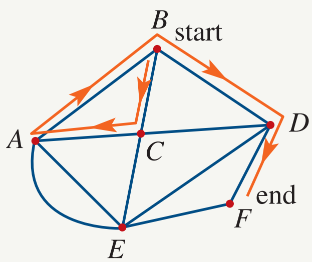
a



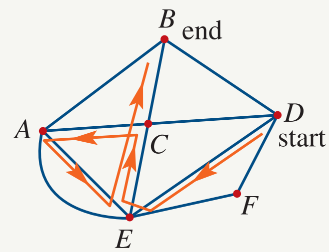
b



c



d

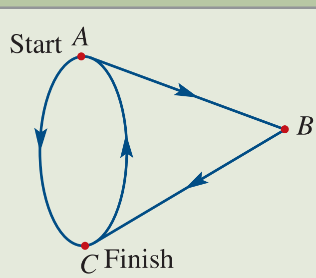
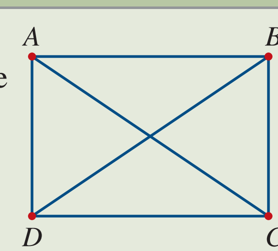


SOLUTION:

1. This walk starts and ends at the same vertex without repeated edges so it is either a circuit or a cycle. The walk passes through vertex C twice without repeated edges, so it must be a circuit. — **a** Circuit
2. This walk starts and ends at the same vertex with no repeated edges so it is either a circuit or a cycle. The walk has no repeated vertex so it is a cycle. — **b** Cycle
3. This walk starts at one vertex and ends at a different vertex, so it is not a circuit or a cycle. It has one repeated vertex (B) and no repeated edge, so it must be a trail. — **c** Trail
4. This walk starts at one vertex and ends at a different vertex so it is not a circuit or a cycle. It has repeated vertices (C and E) and repeated edges (the edge between C and E), so it must be a walk only. — **d** Walk only

Traversable graphs

Many practical problems involve finding a trail in a graph that includes every edge. You can trace out a trail on the graph without repeating an edge or taking the pen off the paper. Graphs that have this property are called traversable graphs. They will be met again in section 2D.

Traversable graph	Non-traversable graph
A traversable graph has a trail that includes every edge. The trail A–B–C–A–C is one example. 	Not all graphs are traversable. It is impossible to find a trail in a non-traversable graph that includes every edge. 

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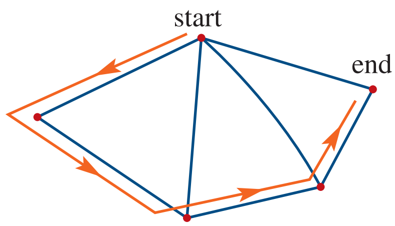
Exercise 2B

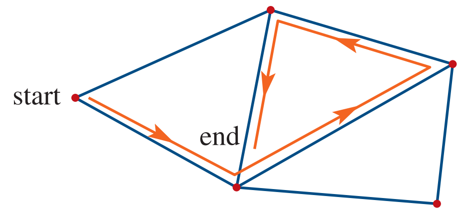
Example 2

1 Identify the walk in each of the graphs below as a trail, path, circuit or walk only.

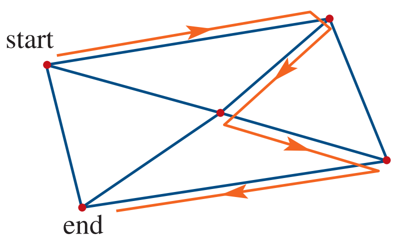
a



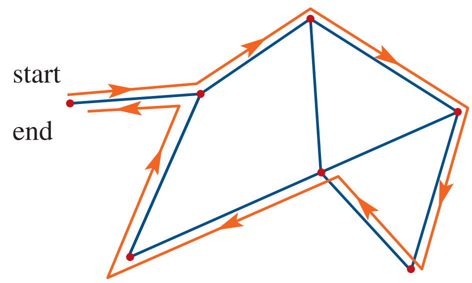
b



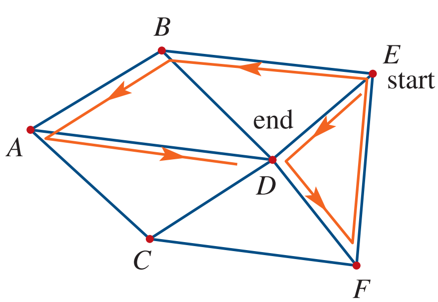
c



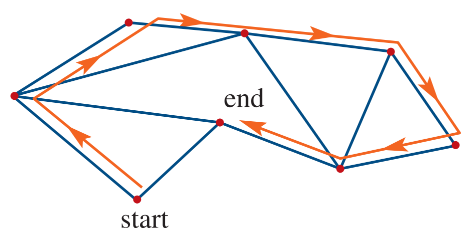
d



e



f



2 Using the graph below, identify the walks below as a trail, path, circuit, cycle or walk only.

a *A–B–E–B–F*

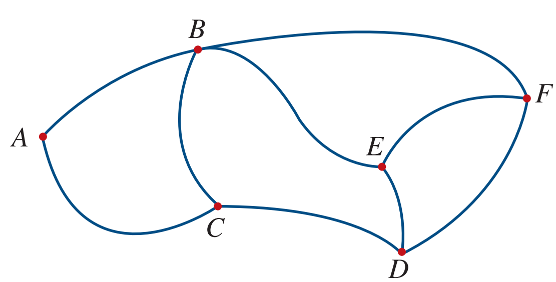
b *B–C–D–E–B*

c *C–D–E–F–B–A*

d *A–B–E–F–B–E–D*

e *E–F–D–C–B*

f *C–B–E–F–D–E–B–C–A*



3 Identify the following sequence of vertices as either a trail or a cycle.

a *A–C–B–D–A*

b *P–R–S–Q*

c *M–N–O–P–M*

d *C–B–E–A–F–E–G–D*

e *D–E–A–F–C–B–D*

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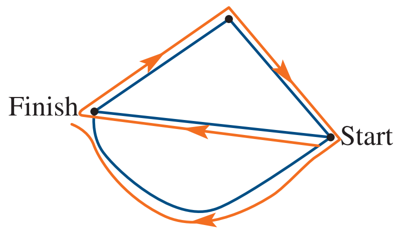
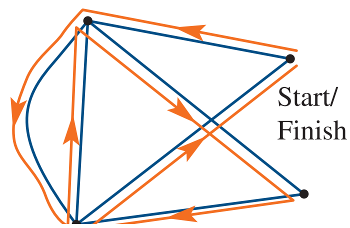
4 Identify the walk in each of the graphs below as a circuit or trail.

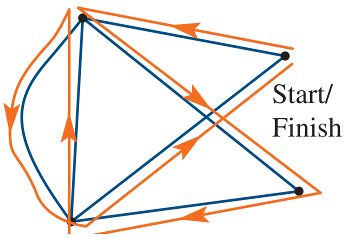
a



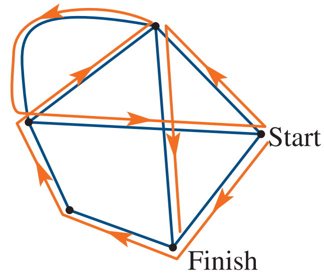
b



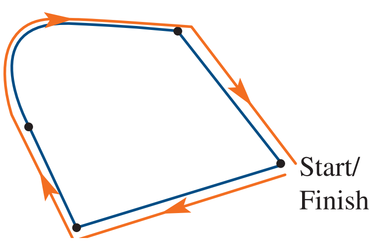
c



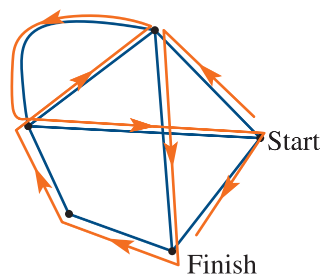
d



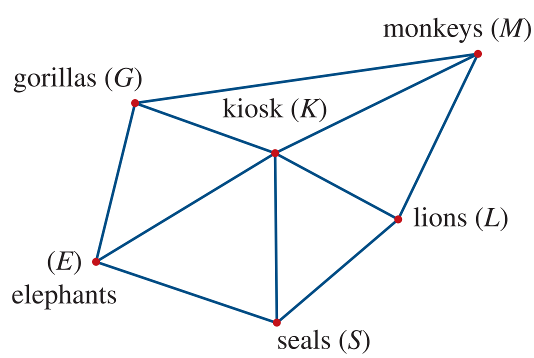
e



f



5 The network diagram below shows the pathway linking five animal enclosures in a zoo to each other and to the kiosk.



a Which of the following represents a trail?

- **i** S-L-K-M-K
- **ii** G-K-L-S-E-K-M
- **iii** E-K-L-K

b Which of the following represents a path?

- **i** K-E-G-M-L
- **ii** E-K-L-M
- **iii** K-H-E-K-G-M

c Which of the following represents a circuit?

- **i** K-E-G-M-K-L-K
- **ii** E-S-K-L-M-K-E
- **iii** K-S-E-K-G-K

d Which of the following represents a cycle?

- **i** K-E-G-K
- **ii** G-K-M-L-K-G
- **iii** L-S-E-K-L

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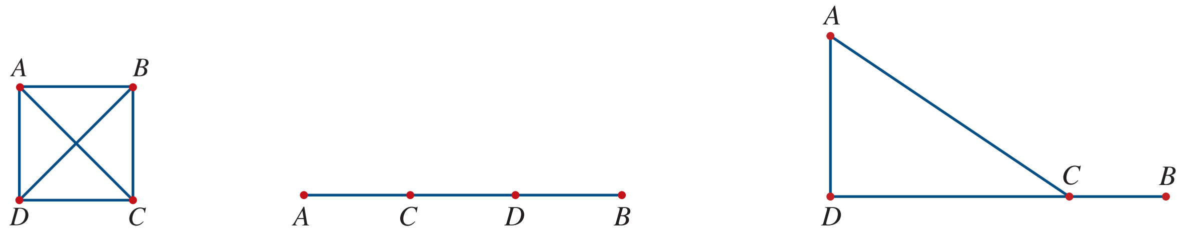
2C Drawing a network diagram

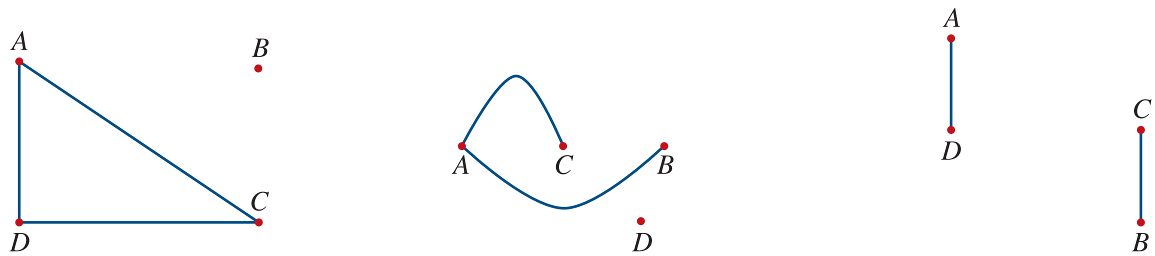
There are many situations in everyday life that involve connections between people or objects. Towns are connected by roads, computers are connected to the internet and families are connected to each other. The network diagram opposite demonstrates some of the connections on social media. When constructing a network, the graphs are either connected or not connected.

Connected graphs

A connected graph has every vertex connected to every other vertex, either directly or indirectly via other vertices. That is, every vertex in the graph can be reached from every other vertex in the graph. The three graphs shown below are all connected.



The graphs are connected because, starting at any vertex, say A, you can always find a path along the edges of the graph to take you to every other vertex. However, the three graphs below are not connected, because there is not a path along the edges that connects vertex A (for example) to every other vertex in the graph.



CONNECTED GRAPH

A graph is connected if every vertex in the graph is accessible from every other vertex in the graph along a path formed by the edges of the graph.

Isomorphic graphs

Different looking graphs can contain the same information. When this happens, we say that these graphs are equivalent or isomorphic. For example, the following three graphs look quite different but, in graphical terms, they are equivalent.

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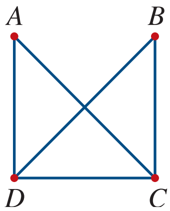
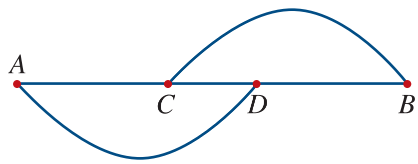
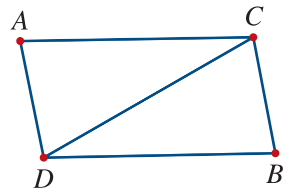
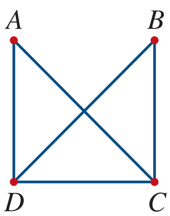
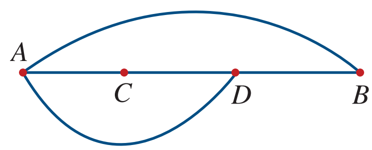


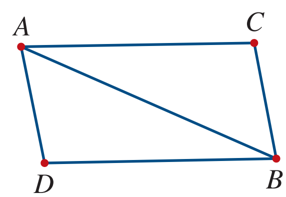




Each of the above graphs has the same number of edges (5), vertices (4), the corresponding vertices have the same degree and the edges join the vertices in the same way (*A* to *C*, *A* to *D*, *B* to *C*, *B* to *D*, and *D* to *C*). However, the three graphs below, although having the same numbers of edges and vertices, are not isomorphic. This is because corresponding vertices do not have the same degree and the edges do not connect the same vertices







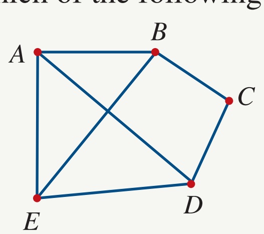
ISOMORPHIC GRAPHS

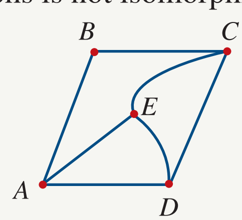
Two graphs are isomorphic (equivalent) if:

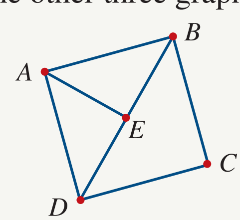
- they have the same numbers of edges and vertices
- corresponding vertices have the same degree and the edges connect to the same vertices.

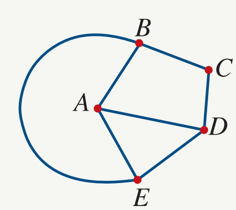
Example 3: Identifying an isomorphic graph 2C

Which of the following graphs is not isomorphic to the other three graphs?

1


2


3


4


SOLUTION:

1 Check that each graph has the same number of vertices and edges.

Every graph has five vertices and seven edges.

2 Check that corresponding vertices have the same degree.

In Graph 2, vertex *B* has degree 2 and *C* has degree 3; in all others, *B* has degree 3 and *C* has degree 2.

3 Check that edges connect to the same vertices.

In graphs 1, 3 and 4, the edges are *A*–*B*, *A*–*D*, *A*–*E*, *B*–*C*, *B*–*E*, *C*–*D* and *D*–*E*, so these graphs are isomorphic.
Graph 2 does not have edge *B*–*E* and does have edge *C*–*E*, which does not appear in the other graphs, showing again that it is not isomorphic to the others.
Hence, graph 2 cannot be isomorphic to any of the other graphs shown.

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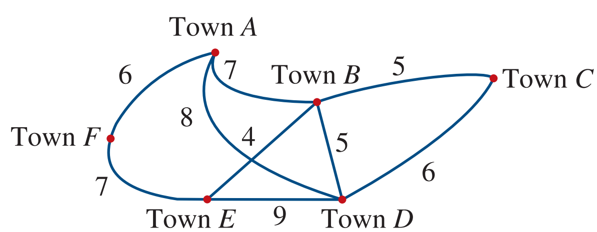
Weighted graphs

The edges of graphs represent connections between the vertices. Sometimes there is more information known about that connection. If the edge of a graph represents a road between two towns, we might also know the length of this road, or the time it takes to travel this road. Extra numerical information about the edge that connects vertices can be added to a graph by writing the number next to the edge. This is called a weighted edge. Graphs that have a number associated with each edge are called weighted graphs.

WEIGHTED GRAPH

A weighted graph is a network diagram that has weighted edges or an edge with number assigned to it that implies some numerical value such as cost, distance or time.

The weighted graph in the diagram on the right shows towns, represented by vertices, and the roads between those towns, represented by edges. The numbers, or weights, on the edges are the distances along the roads. A problem often presented by this road network is, 'What is the shortest distance between certain towns?'



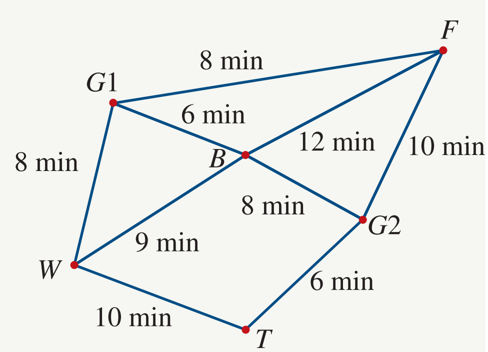
While this question is easily answered if all the towns are directly connected such as Town A to Town B, the answer is not so obvious if we have to travel through towns to get there such as Town F to Town C.

Example 4: Solving a practical network problem 2C

The network diagram opposite is used to model the tracks in a forest connecting a suspension bridge(B), a waterfall (W), a very old tree (T) and a fern gully (F). Walkers can enter or leave the forest through either gate 1 ($G1$) or gate 2 ($G2$). The numbers on the edges represent the times (in minutes) taken to walk directly between these places.

a How long does it take to walk from the bridge directly to the fern gully?

b How long does it take to walk from the old tree to the fern gully via the waterfall and the bridge?



SOLUTION:

1 Identify the edge that directly links the bridge with the fern gully and read off the time.

a The edge is $B–F$.
The time taken is 12 minutes.

2 Identify the path that links the old tree to the fern gully, visiting the waterfall and the bridge on the way. Add up the times.

b The path is $T–W–B–F$.
Time = 10 + 9 + 12 = 31.
The time taken is 31 minutes.

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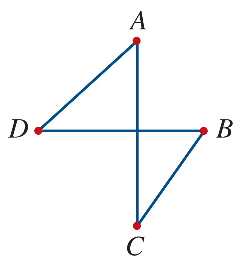
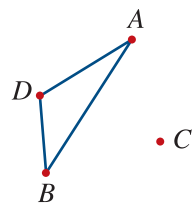
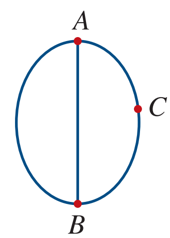
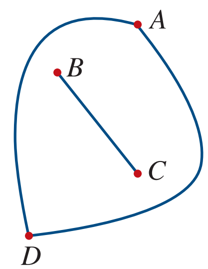
Exercise 2C

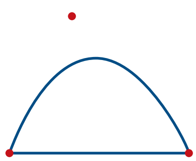
1 Which of the following graphs are connected in each question?

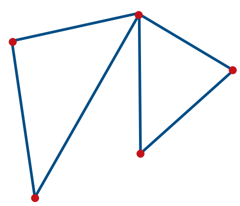
a Graph 1 

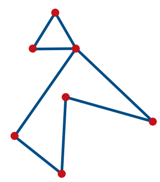
Graph 2 

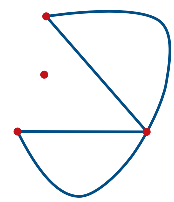
Graph 3 

Graph 4 

b Graph 1 

Graph 2 

Graph 3 

Graph 4 

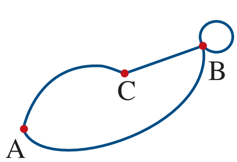
2 Draw a connected graph with:

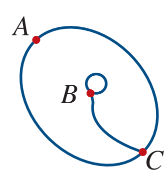
a three vertices and three edges
b three vertices and five edges
c four vertices and six edges
d five vertices and five edges.

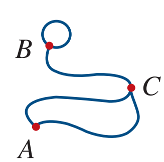
3 Draw a graph that is not connected with:

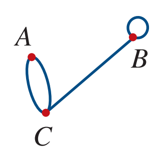
a three vertices and two edges
b four vertices and three edges
c four vertices and four edges
d five vertices and three edges.

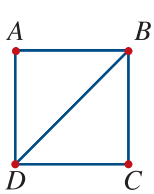
Example 3 **4** Which of the following graphs is not isomorphic to the other three graphs in each question?

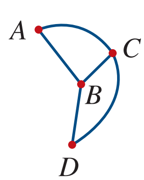
a Graph 1 

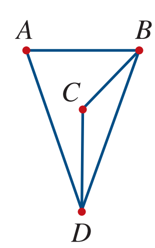
Graph 2 

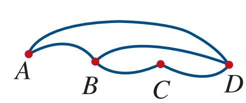
Graph 3 

Graph 4 

b Graph 1 

Graph 2 

Graph 3 

Graph 4 

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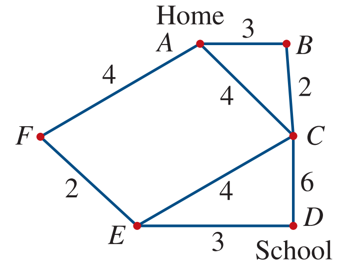
Example 4 **5** Evelyn has drawn a network diagram to represent several streets for travelling from her home to school. The numbers indicate the times in minutes.



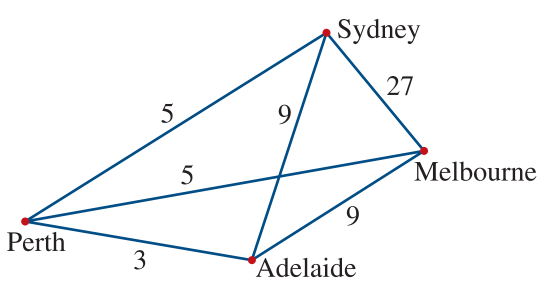
a How long does it take to walk from home to school using the following paths?
   i *A–F–E–D*
   ii *A–F–E–C–D*
   iii *A–C–D*
   iv *A–B–C–D*
   v *A–C–E–D*

b Which of the above walks is the longest journey?

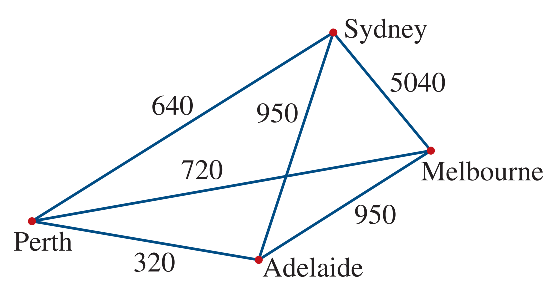
c Which of the above walks is the shortest journey?

6 The network graph below shows details about air travel between Australian cities. The first graph shows the number of flights in each direction between cities, and the second graph shows capacity in each direction.

Number of flights per day



Maximum number of passengers per day



a How many flights are available per day from Sydney to Adelaide?

b How many flights are available per day from Sydney to Melbourne?

c How many flights are available per day between these Australian cities?

d What is the maximum number of passengers per day from Melbourne to Adelaide?

e What is the maximum number of passengers per day from Sydney to Perth?

f What is the maximum number of passengers per day from Melbourne to Perth?

g What is the maximum number of passengers per day that can fly out of Sydney?

h Seth is wishing to fly from Perth to Melbourne but is told that the direct flights are fully booked. List any other ways of completing this journey.

i What is the greatest number of people per day that can be flown from Perth to Melbourne?

j Amy is wishing to fly from Sydney to Melbourne but is told that the direct flights are fully booked. List any other ways of completing this journey.

k What is the greatest number of people per day that can be flown from Sydney to Melbourne?

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2D Eulerian and Hamiltonian walks

Eulerian trails and circuits

An Eulerian trail is a trail that uses every edge of a graph exactly once. Eulerian trails start and end at different vertices. Similarly, an Eulerian circuit is a circuit that uses every edge of a graph exactly once. Eulerian circuits start and end at the same vertex. Eulerian trails and circuits are important for some real-life applications. For example, if a graph shows towns as vertices and roads as edges, then being able to identify a route through the graph that follows every road is important for mail delivery. A graph with an Eulerian trail is an example of a traversable graph.

An Eulerian trail will exist if the graph is connected and has exactly two vertices with an odd degree. These two vertices of odd degree will form the start and end of the Eulerian trail. Eulerian circuits will exist if every vertex of the graph has an even degree. These results were discovered by the Swiss mathematician called Leonhard Euler.

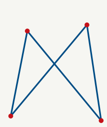
EULERIAN TRAIL	EULERIAN CIRCUIT
A trail that uses every edge of a graph exactly once and starts and ends at different vertices. Eulerian trails exist if the graph has exactly two vertices with an odd degree.	A circuit that uses every edge of a network graph exactly once, and starts and ends at the same vertex. Eulerian circuits exist if every vertex of the graph has an even degree.

Eulerian trails and circuits Watch the video in the Interactive Textbook to see them in action.

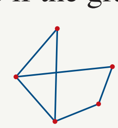
Example 5: Identifying Eulerian trails and circuits 2D

For each of the following graphs, determine whether the graph has an Eulerian trail, an Eulerian circuit or neither. Show one example if the graph has an Eulerian trail or Eulerian circuit.

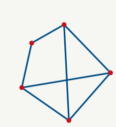
a



b



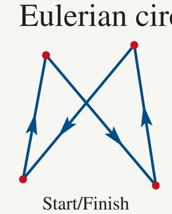
c



SOLUTION:

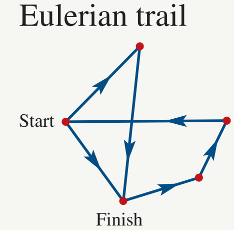
1 All the vertices in the graph have an even degree (degree 2).
2 Even degree indicates there is an Eulerian circuit.
3 Start and finish from the vertex on the bottom left-hand side and travel through each edge once.

a Eulerian circuit



4 Two of the vertices in the graph have an odd degree (degree 3) and the remaining vertices have an even degree (degree 2)
5 Two odd degrees indicates there is an Eulerian trail.
6 Start and finish from the vertices with the odd degrees.

b Eulerian trail



7 Four vertices are odd and one is even. No Eulerian trail or circuit.

c Neither

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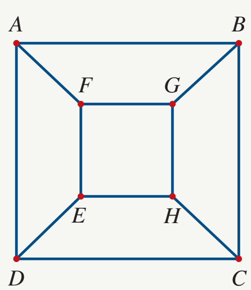
ENRICHMENT: Hamiltonian paths and cycles

Eulerian trails and circuits are focused on the edges (though the degree of the vertices will tell you if walk is Eulerian). Hamiltonian paths and cycles are focused on the vertices. A Hamiltonian path passes through every vertex of a graph once and only once. It may or may not involve all the edges of the graph. A Hamiltonian cycle is a Hamiltonian path that starts and finishes at the same vertex. Hamiltonian paths and cycles have real-life applications where every vertex of a graph needs to be visited, but the route taken is not important. For example, if the vertices of a graph represent people and the edges of the graph represent email connections between those people, a Hamiltonian path would ensure that every person in the graph received the email message. Unlike Eulerian trails and circuits, Hamiltonian paths and cycles do not have a convenient rule or feature that identifies them. Inspection is the only way to identify them. Hamiltonian paths and cycles are named after an Irish mathematician called Sir William Hamilton.

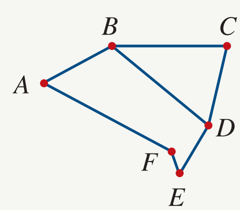
HAMILTONIAN PATH	HAMILTONIAN CYCLE
A path passes through every vertex of a graph once and only once.	A Hamiltonian path that starts and finishes at the same vertex.

Example 6: Identifying a Hamiltonian path and cycle — 2D

a List a Hamiltonian path for the network graph below that starts at A and finishes at D.



b Identify a Hamiltonian cycle for the network graph below starting at A.



SOLUTION:

1. A Hamiltonian path involves all the vertices but not necessarily all the edges.
2. The solution $A–F–G–B–C–H–E–D$ is not unique. There are other solutions such as $A–F–E–H–G–B–C–D$.
3. A Hamiltonian circuit is a Hamiltonian path that starts and finishes at the same vertex.
4. The solution $A–B–C–D–E–F–A$ is not unique. There are other solutions such as $A–F–E–D–C–B–A$.

a $A–F–G–B–C–H–E–D$

b $A–B–C–D–E–F–A$

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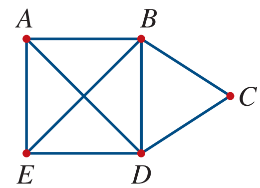
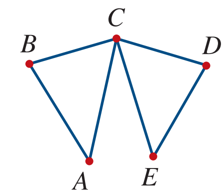
Exercise 2D

1 A network graph is shown opposite.
  a What is the degree of each vertex?
  b Why does this graph have an Eulerian trail?
  c List the Eulerian trail.

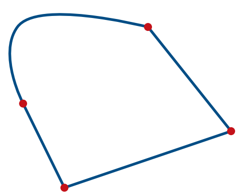


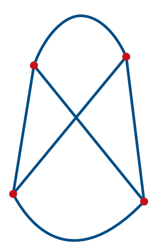
2 A network graph is shown opposite.
  a What is the degree of each vertex?
  b Why does this graph have an Eulerian circuit?
  c List an Eulerian circuit.



Example 5

3 For each of the following graphs, determine whether the graph has an Eulerian trail, an Eulerian circuit or neither. Show one example if the graph has an Eulerian trail or circuit.

a


b


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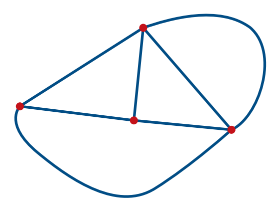

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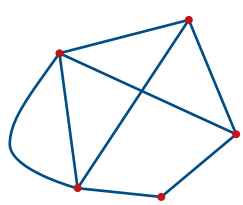

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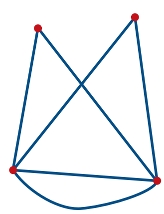

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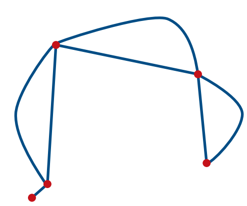
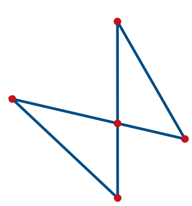


g


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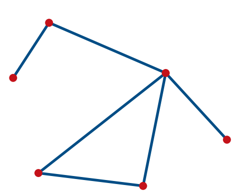
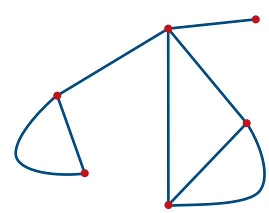
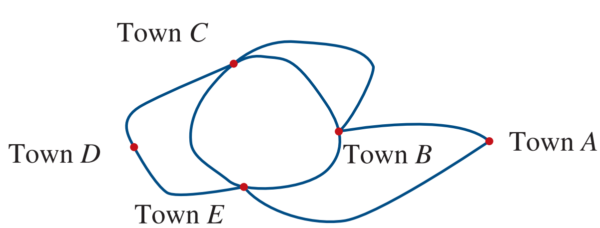


i


4 A road inspector lives in town *A* and is required to inspect all roads connecting the neighbouring towns *B*, *C*, *D* and *E*.
  a Is it possible for the inspector to travel over every road linking the five towns only once and return to town *A*? Explain.
  b Show one possible route he can follow.



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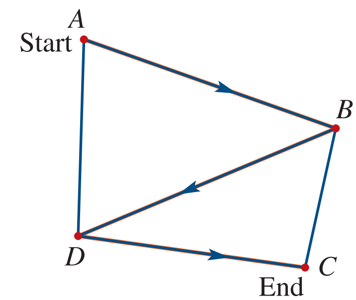
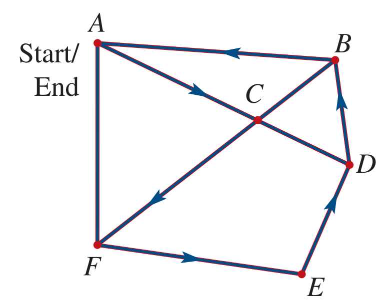
5 A network graph is shown opposite.

a List the path shown in the graph.
b Does the path pass through every vertex?
c Does the path pass through every edge?
d ENRICHMENT: Why is this path a Hamiltonian path?
e ENRICHMENT: List another Hamiltonian path starting at *A*.



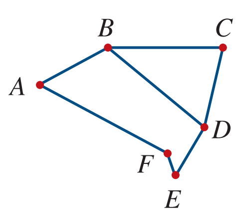
6 A network graph is shown opposite.

a List the path shown in the graph.
b Does the path pass through every vertex?
c Does the path pass through every edge?
d ENRICHMENT: Why is this path a Hamiltonian cycle?
e ENRICHMENT: List another Hamiltonian cycle starting at *A*.



Example 6

7 ENRICHMENT: List a Hamiltonian cycle for each of the following.

a


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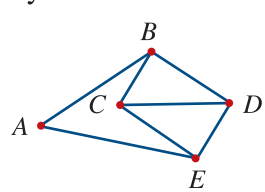
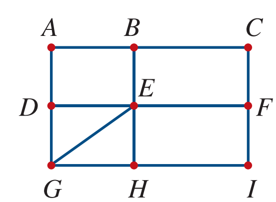


c


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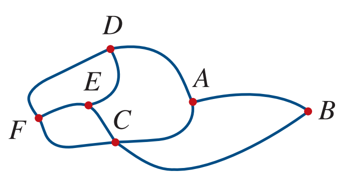

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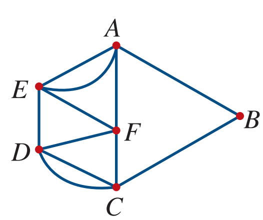

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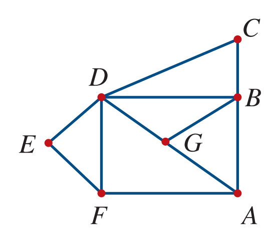
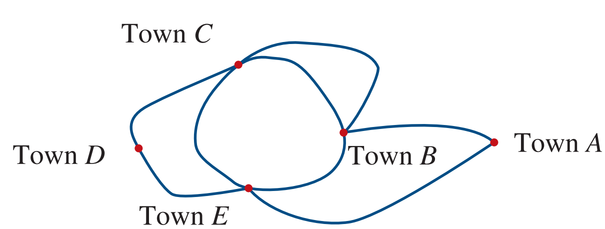


8 ENRICHMENT: A tourist wants to visit each of five towns shown in the graph opposite only once. Identify one possible route for the tourist to start the tour at:

a *C* and finish at *A*. What is the mathematical name for this route?
b *E* and finish at *E*. What is the mathematical name for this route?



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2E Network problems

Example 7: Drawing a network diagram to represent a map 2E

The map shows the main highways between the capital cities of the states and territories of Australia. Construct a network diagram of the main highway connections between the cities. Let each capital city be a vertex and the highway route between the cities an edge.

SOLUTION:

1 Vertices are the dots of the network diagram. In this situation the capital city will be a vertex.

The vertices will be Brisbane, Sydney, Canberra, Melbourne, Hobart, Adelaide, Perth and Darwin.

2 Edges are the connections or pathways between the vertices of the network diagram.

Highway routes connect the cities with the exception of Hobart.

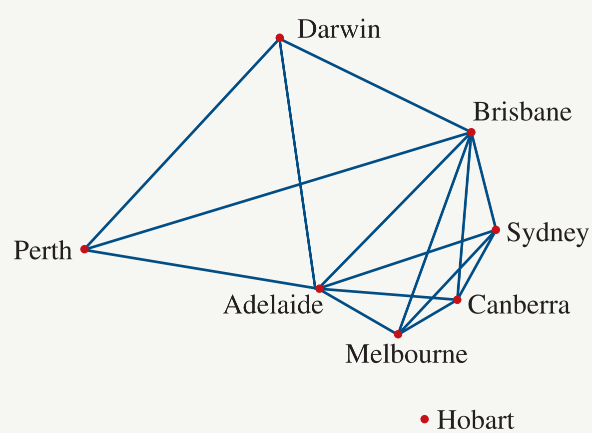
3 Start by drawing a dot for each vertex (capital city).

4 Label each vertex with the name of the capital city.

5 Draw a line to represent an edge if the capital cities have a highway route between them

6 The network diagram is not connected, as Hobart does not have a highway linking it to any other city.

7 The proportions of the network diagram do not have to match the real-life situation.



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Example 8: Drawing a network diagram to represent a table 2E

A group of four students worked in pairs on four different problems. The table below shows the problem number and the two students who found the correct solution to that problem. A network diagram is to be constructed to represent the table.

Problem	Students who solved it	
1	Darcy	Beau
2	Alyssa	Beau
3	Alyssa	Claire
4	Darcy	Claire

a What will be the vertices of the network diagram?
b What will be an edge in the network diagram?
c Draw a network diagram to represent the information in the table.
d Draw an isomorphic graph of the network diagram.
e Which students have not been able to solve a problem together?

SOLUTION:

1 Vertices are the dots of the network diagram. In this situation the student will be a vertex.

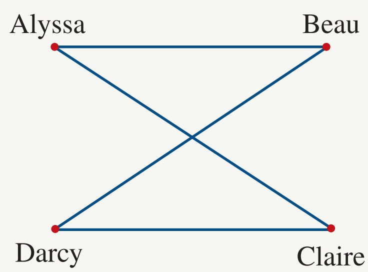
a Alyssa, Beau, Claire and Darcy.

2 Edges are the connections or pathways between the vertices of the network diagram.

b Edges are drawn if the students have worked together to solve the problem.

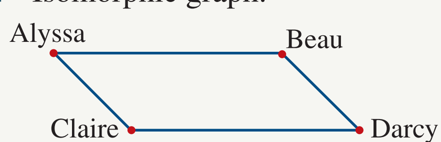
3 Start by drawing a dot for each vertex (student).
4 Label each vertex with the student's name.
5 Draw a line to represent an edge if the students have worked together to solve the problem.
6 The network diagram is connected since the path along the edges can take you to every other vertex.

c Network diagram



7 Isomorphic graphs have the same number of vertices and edges. They must also have to show the same connections.

d Isomorphic graph.



8 Vertices not connected with an edge represent the students who have not been able to solve a problem.

e Alyssa and Darcy, Beau and Claire

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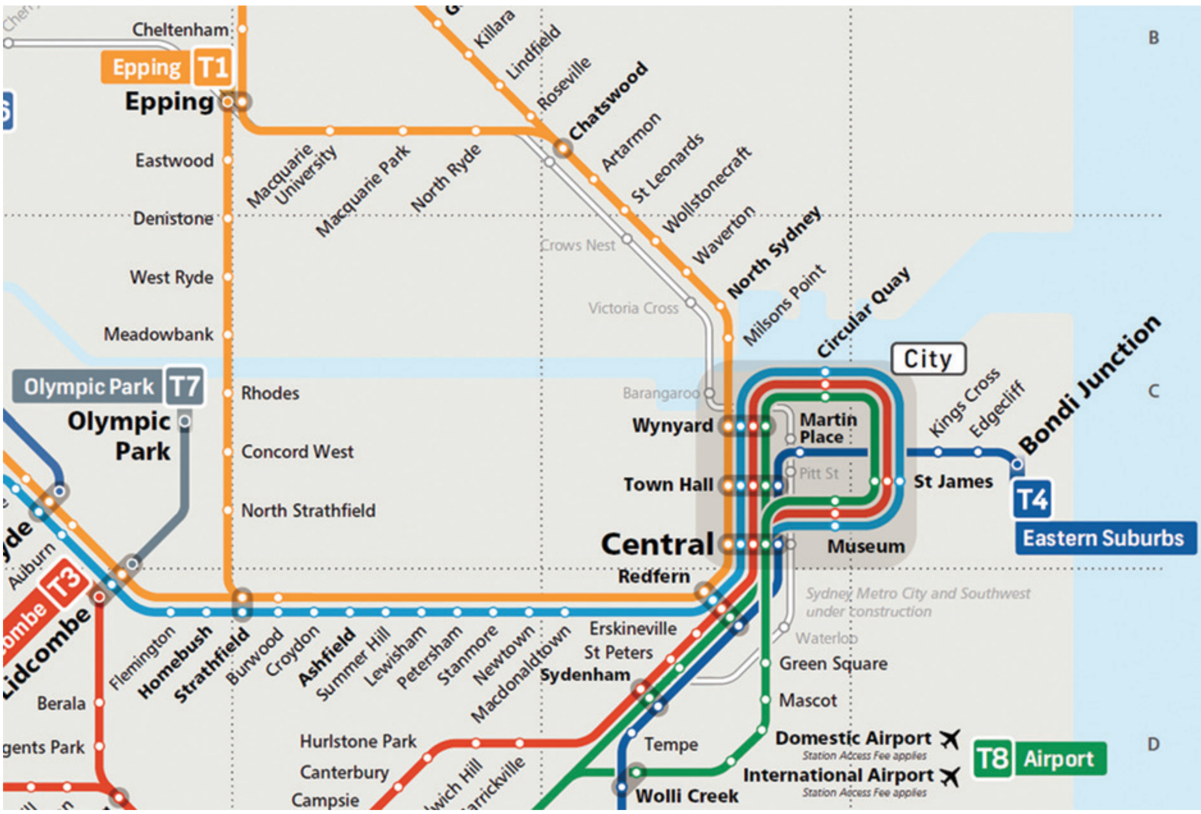
Exercise 2E

1 This is a network diagram for Sydney trains.

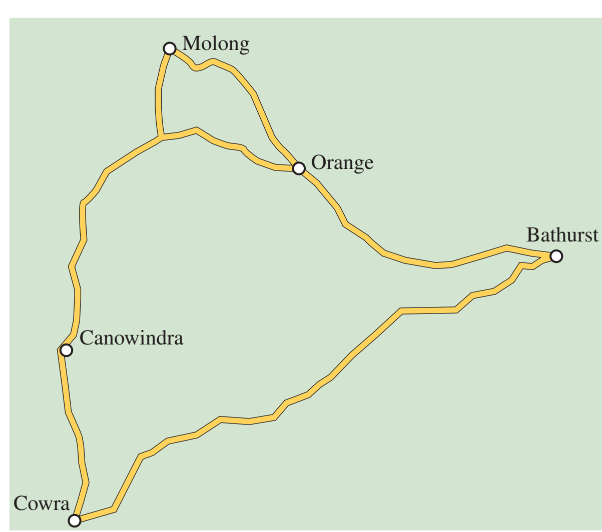


a What are the vertices of the network diagram?
b What are the edges in the network diagram?
c Is this network diagram connected?
d How many vertices are there in the city circle (shaded circle)?
e Draw a network diagram to represent the city circle.

Example 7

2 A map from a NSW region is shown on the right.

a What are the vertices of the network diagram?
b What are the edges in the network diagram?
c Is this network diagram connected?
d Draw an isomorphic graph to this network diagram.



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Example 8 **3** Four friends live close to each other. The table opposite shows the friends and the number of minutes to walk between their homes.

a Draw a network diagram to represent the information in the table.

b What are the vertices of the network diagram?

c What does a weighted edge represent in the network diagram?

d Which friends do not have a direct path between their homes?

e What is the shortest total walking time for Alex to leave home and visit Zac, Max and Harvey in that order, and to return home. Ignore any time spent in each house.

Friends		Minutes to walk between homes
Alex	Zac	1
Harvey	Zac	3
Max	Zac	2
Harvey	Max	4
Alex	Harvey	2

4 There are six motorways between six cities labelled A, B, C, D, E and F. The table opposite shows which cities are linked by the motorways and the length of each one in kilometres.

a Draw a network diagram to represent the information in the table.

b What are the vertices of the network diagram?

c What does a weighted edge represent in the network diagram?

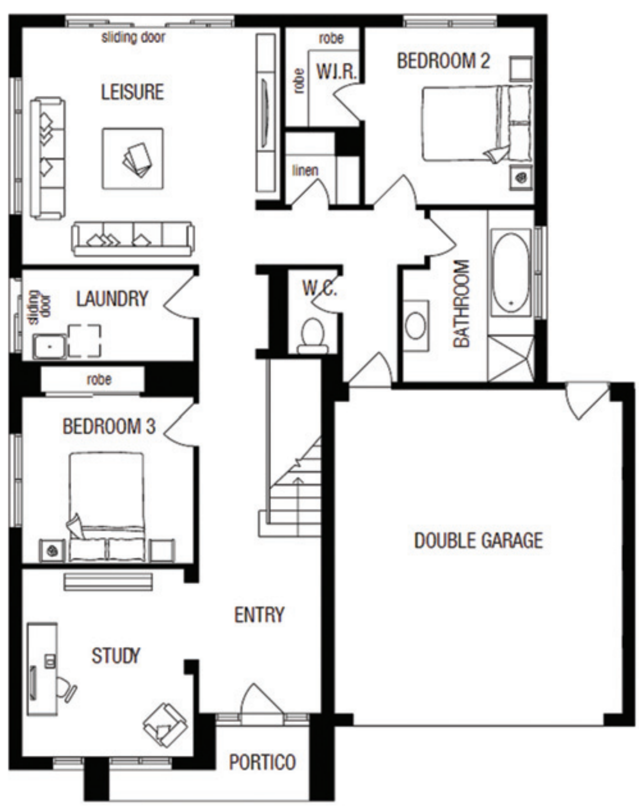
d Which cities are not directly linked to city A?

e How would you travel from city F to city D?

f What is the shortest journey between city F and city D?

	A	B	C	D	E	F
A	–	–	–	27	51	35
B	–	–	48	24	–	–
C	–	48	–	12	–	–
D	27	24	12	–	–	–
E	51	–	–	–	–	–
F	35	–	–	–	–	–

5 The first floor plan of a house is shown opposite. Draw a network diagram by letting the rooms be the vertices and the doorways the edges. Make the hall a vertex.



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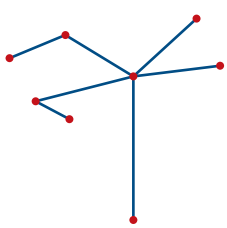
2F Minimal spanning trees

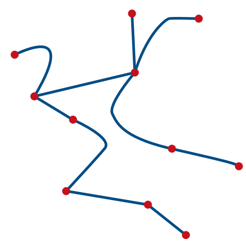
In some network problems it is important to minimise the number and weights of the edges to keep all vertices connected to the graph. For example, a number of towns might need to be connected to a water supply. The cost of connecting the towns can be minimised by connecting each town into a network or water pipes only once, rather than connecting each town to every other town. To solve these problems we need an understanding of trees.

Tree

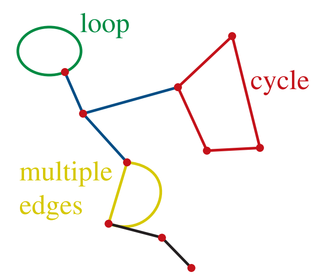
A tree is a connected graph that contains no cycles, multiple edges or loops. The diagram below shows two network graphs that are trees and one network graph that is not a tree.



Graph 1: a tree



Graph 2: a tree



Graph 3: not a tree

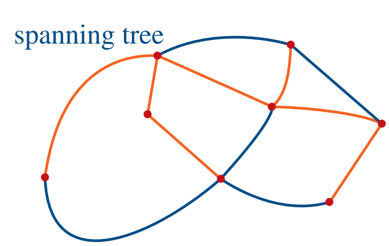
Graphs 1 and 2 are trees: they are connected and have no cycles, multiple edges or loops. Graph 3 is not a tree: it has several cycles (loops and multiple edges count as cycles). For trees, there is a relationship between the number of vertices and the number of edges.

- Graph 1, a tree, has 8 vertices and 7 edges.
- Graph 2, a tree, has 11 vertices and 10 edges.

In general, the number of edges is always one less than the number of vertices. In other words, a tree with n vertices has $n - 1$ edges.

Spanning trees

Every connected graph will have at least one subgraph that is a tree. If a subgraph is a tree, and if that tree connects all of the vertices in the graph, then it is called a spanning tree. An example of a spanning tree is shown opposite. There are several other possibilities. Note: the spanning tree opposite, like the network diagram, has 8 vertices. However it has only 7 edges ($8 - 1 = 7$).



TREE

A tree is a connected graph that contains no cycles, multiple edges or loops.
A tree with n vertices has $n - 1$ edges.
A spanning tree is a tree that connects all of the vertices of a graph.

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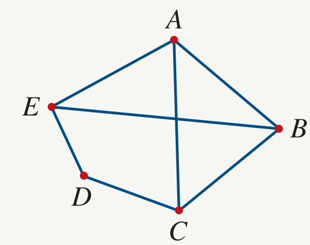
Example 9: Finding a spanning tree in a network 2F

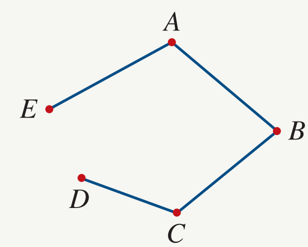
Find two spanning trees for the graph shown opposite.



SOLUTION:

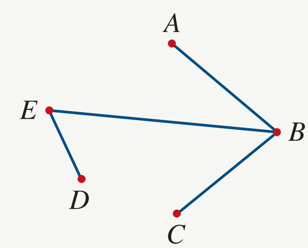
1. The graph has five vertices and seven edges. A spanning tree will have five (n) vertices and four ($n - 1$) edges.
2. To form a spanning tree, remove any three edges, provided that all the vertices remain connected, and there are no multiple edges or loops.

Spanning tree 1



3. Spanning tree 1 is formed by removing edges *EB*, *ED* and *CA*.
4. Spanning tree 2 is formed by removing edges *EA*, *AC* and *CD*.
5. There are several other possible spanning trees.

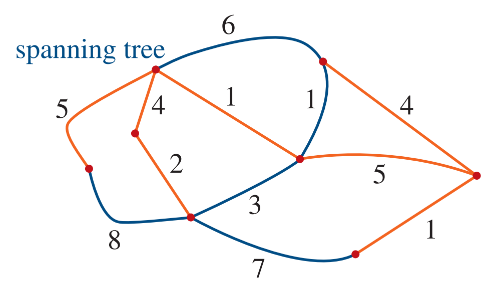
Spanning tree 2



Minimum spanning trees

For a weighted graph, it is possible to determine the 'length' of each spanning tree by adding up the weights of the edges in the tree. For the spanning tree opposite:

Length $= 5 + 4 + 2 + 1 + 5 + 4 + 1$
$= 22$ units



A minimum spanning tree is a spanning tree of minimum length. It connects all the vertices together with the minimum total weighting for the edges.

MINIMUM SPANNING TREE

A minimum spanning tree is a spanning tree of minimum length. It connects all the vertices together with the minimum total weighting for the edges.

A guide to trees Watch the video in the Interactive Textbook to see trees, spanning trees and minimum spanning trees in action.

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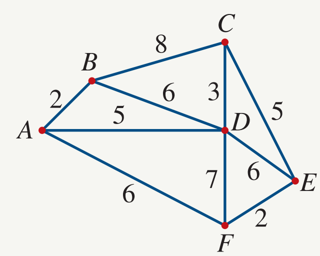
Prim's algorithm

Prim's algorithm is a set of rules to determine a minimum spanning tree for a graph.

1. Choose a starting vertex (any will do).
2. Inspect the edges starting from the starting vertex and choose the one with the lowest weight. (If there are two edges that have the same weight, it does not matter which one you choose). You now have two vertices and one edge.
3. Inspect all of the edges starting from both of the vertices you have in the tree so far. Choose the edge with the lowest weight, ignoring edges that would connect the tree back to itself. You now have three vertices and two edges.
4. Keep repeating step 3 until all of the vertices are connected.

Example 10: Finding the minimum spanning tree — 2F

Apply Prim's algorithm to find the minimum spanning tree for the network graph shown on the right. Calculate the length of the minimum spanning tree.



SOLUTION:

1 Start with vertex A. List the weighted edges from vertex A and find the smallest.

$(A, B) = 2$
$(A, F) = 6$
$(A, B) = 2$ is lowest.

2 Look at vertices A and B. List the weighted edges from vertex A and vertex B (apart from (A, B) which you have already found).

$(A, D) = 5$
$(A, F) = 6$
$(B, C) = 8$
$(B, D) = 6$
$(A, D) = 5$ is lowest.

3 Repeat to find the smallest weighted edge from vertex A, B or D.

$(C, D) = 3$ is lowest.

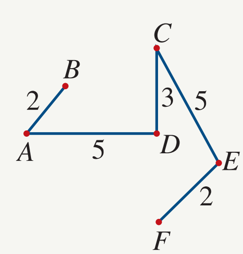
4 Repeat to find the smallest weighted edge from vertex A, B, D or C.

$(C, E) = 5$ is lowest.

5 Repeat to find the smallest weighted edge from vertex A, B, D, C or E.

$(E, F) = 2$ is lowest.

6 All vertices have been included in the graph. Draw the minimum spanning tree.



7 Find the length of the minimum spanning tree by adding the weights of the edges.

Length $= 2 + 5 + 3 + 5 + 2$
$= 17$ units

Kruskal's algorithm

Kruskal's algorithm This alternative method is covered in the Interactive Textbook.

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Exercise 2F

1 Copy and complete the following sentences.

a A tree is a connected graph that contains no cycles, multiple __________ or loops.

b A minimum spanning tree is a spanning tree of minimum _________.

c Prim's algorithm is a set of rules to determine a minimum ___________ tree for a graph.

d A connected graph has eight vertices. Its spanning tree has ___________ vertices.

e A connected graph has 10 vertices. Its spanning tree has ___________ edges.

2 **a** How many edges are there in a tree with 15 vertices?

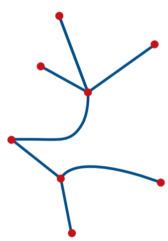
b How many vertices are there in a tree with five edges?

c Draw two different trees with four vertices.

d Draw three different trees with five vertices.

3 Which of the following graphs are trees?

a



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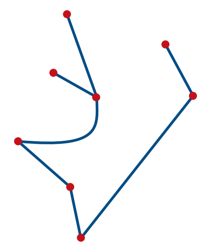
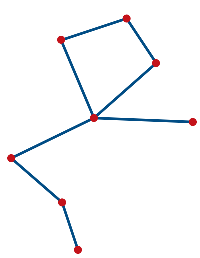


c



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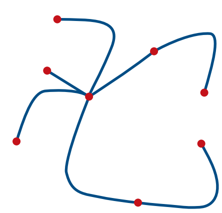

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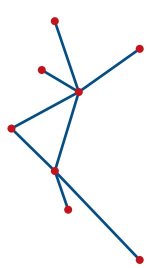

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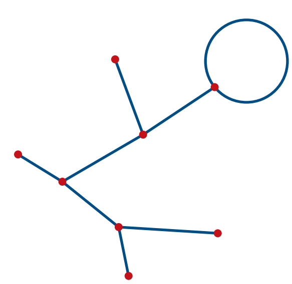
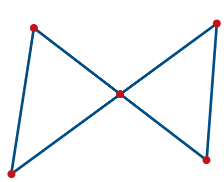


Example 9

4 For each of the following graphs, draw three different spanning trees.

a



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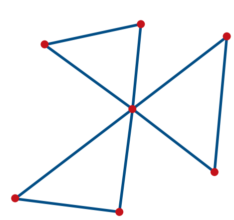
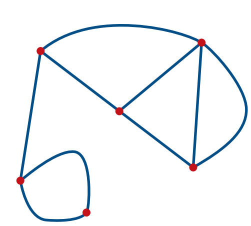


c



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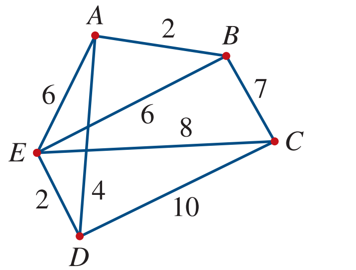
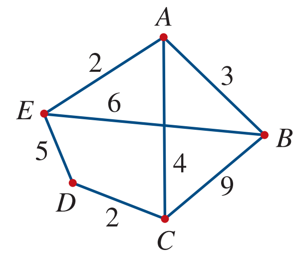
Example 10

5 Find the minimum spanning tree and its length, for each of the following graphs.

a


b


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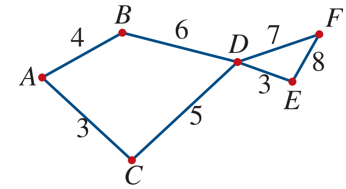
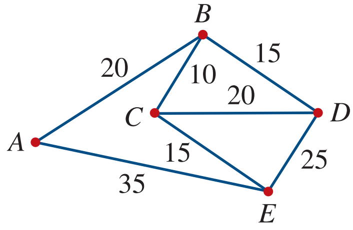
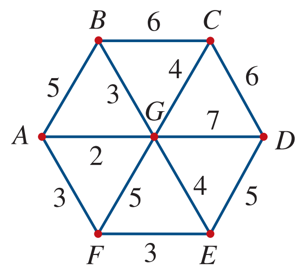
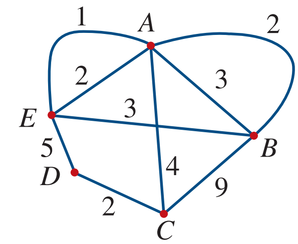
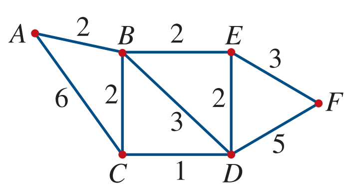
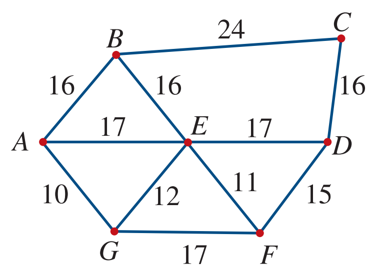


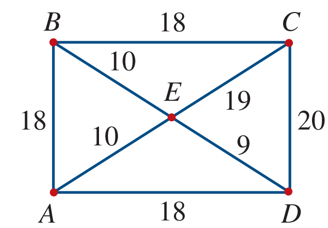
d


e


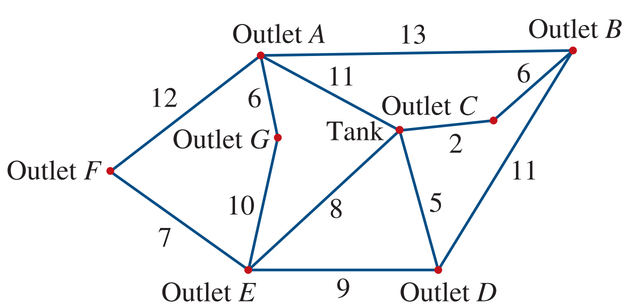
f


g


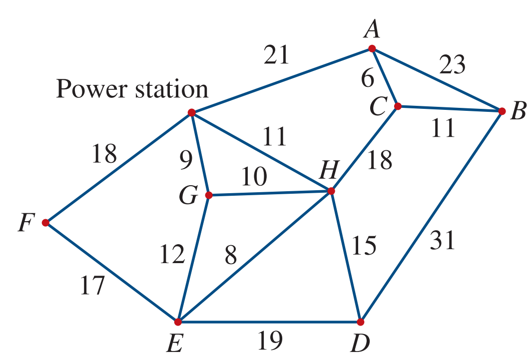
h


i


6 Water is to be piped from a water tank to seven outlets on a property. The distances (in metres) of the outlets from the tank and from each other are shown in the network opposite. Starting at the tank, determine the minimum length of pipe needed.



7 Power is to be connected by cable from a power station to eight substations (*A* to *H*). The distances (in kilometres) of the substations from the power station and from each other are shown in the network opposite. Determine the minimum length of cable needed.



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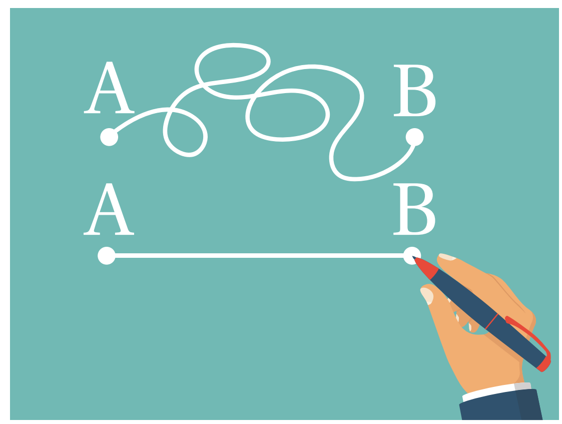
2G Shortest path

The shortest path in a network is the path between two vertices where the sum of the weights of its edges is minimised. Finding the shortest path is often very useful. For example, if the weights of a network represent time, you can choose a path that will allow you to travel in the shortest time. If the weights represent distance, you can determine a path that will allow you to travel the shortest distance. However, be aware that travelling the shortest distance between two places is not necessarily the best path. For example, if shortest path has a speed limit of 60 km/h but another path has a speed limit of 110 km/h then the shortest path may take longer to reach the destination. In such a case, redraw the network diagram with time taken to travel each edge, rather than distance.

While there are sophisticated techniques for solving shortest path problems (such as Dijkstra's algorithm in the next section) the method of inspection involves identifying and comparing the lengths of likely candidates for the shortest path. All of the possible paths should be listed and the length of the path calculated. When finding the shortest path it is important to be aware there can be more than one shortest path between two vertices and the shortest path may not pass through all of the vertices.

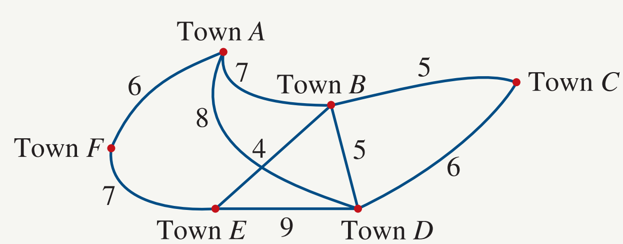


SHORTEST PATH

The shortest path between two vertices in a network is the path where the sum of the weights of its edges is minimised.

Example 11: Finding the shortest path by inspection 2G

Find the shortest path between Town *C* and Town *F*.



SOLUTION:

1. Identify all the likely shortest routes between Town *C* and Town *F* and calculate their lengths.
   Note: Time can be saved by eliminating any route that 'takes the long way around' rather than the direct route. For example, when travelling from Town *B* to Town *D*, ignore the route that goes via Town *A* because it is longer.

$C–D–E–F = 6 + 9 + 7$
$= 22\text{ km}$
$C–B–E–F = 5 + 4 + 7$
$= 16\text{ km}$
$C–B–A–F = 5 + 7 + 6$
$= 18\text{ km}$

2. Compare the different path lengths and identify the shortest path. Write your answer in words.

The shortest path is *C–B–E–F*.

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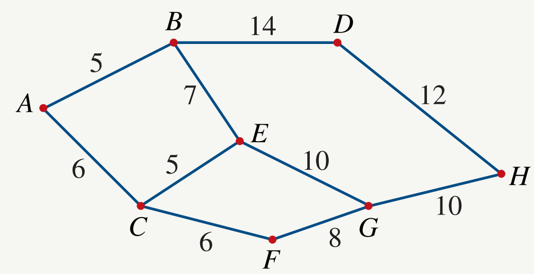
Example 12: Finding the length of the shortest path 2G

Find the length of the shortest path between the following vertices.

a A and E
b A and F
c A and G
d A and H



SOLUTION

1. Identify the shortest paths between vertex A and vertex E.
2. Add the weighted edges to find the length of the path.

a $A–C–E = 6 + 5$
$= 11$

3. Identify the shortest paths between vertex A and vertex F.
4. Add the weighted edges to find the length of the path.

b $A–C–F = 6 + 6$
$= 12$

5. Identify the shortest paths between vertex A and vertex G.
6. Add the weighted edges to find the length of the path.

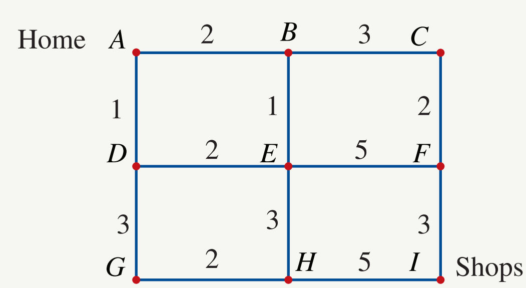
c $A–C–F–G = 6 + 6 + 8$
$= 20$

7. Identify the shortest paths between vertex A and vertex H.
8. Add the weighted edges to find the length of the path.

d $A–C–F–G–H$
$= 6 + 6 + 8 + 10 = 30$

Example 13: Solving a shortest path problem 2G

Darcy has drawn a network diagram to represent several streets for travelling from his home to the shops. The numbers indicate the times in minutes. Describe the shortest path and minimum travelling time.



SOLUTION

1. Identify all the likely shortest routes between home and the shops and calculate their lengths.
2. Compare the different path lengths and identify the shortest path.

$A–D–E–H–I = 1 + 2 + 3 + 5$
$= 11\text{ min}$

$A–D–G–H–I = 1 + 3 + 2 + 5$
$= 11\text{ min}$

$A–B–E–H–I = 2 + 1 + 3 + 5$
$= 11\text{ min}$

$A–B–C–F–I = 2 + 3 + 2 + 3$
$= 10\text{ min}$

3. Write your answer in words.

The shortest path is $A–B–C–F–I$ and the time taken is 10 minutes.

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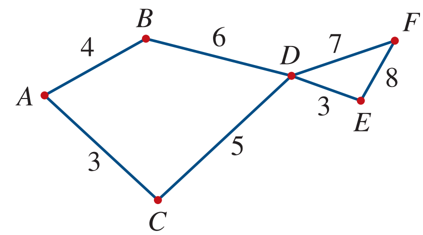
Exercise 2G

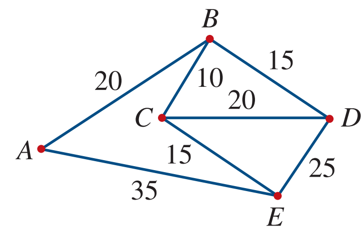
Example 11 **1** The numbers in the weighted graph opposite represent time in hours. Find the length of the shortest path between the following vertices.

a A and D
b A and E
c A and F
d C and F



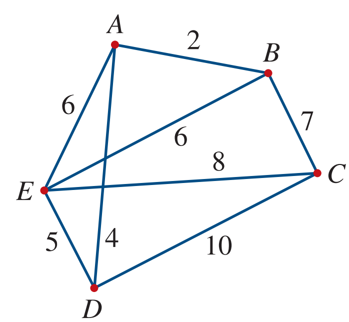
Example 12 **2** The numbers in the weighted graph opposite represent length in metres. Find the length of the shortest path between the following vertices.

a C and D
b A and C
c A and D
d B and E



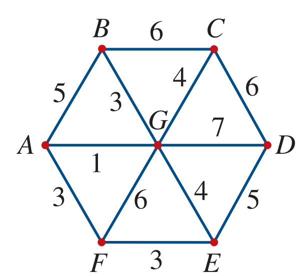
3 The numbers in the weighted graph opposite represent cost in dollars. Find the length of the shortest path between the following vertices.

a E and C **b** B and E
c C and D **d** A and E
e A and C **f** A and D
g B and D **h** A to D to E



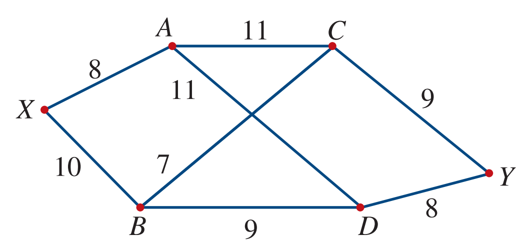
4 The numbers in the weighted graph opposite represent time in minutes. Find the length of the shortest path between the following vertices.

a A and C
b A and E
c B and D
d B and F
e C and F
f C and E



5 The numbers in the weighted graph opposite represent distance in metres. Find the length of the shortest path between the following vertices.

a X and C **b** X and D
c A and Y **d** B and Y
e A and B **f** X and Y



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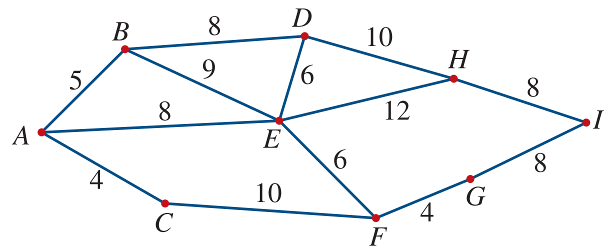
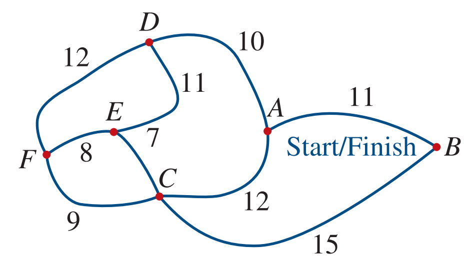
Example 13 **6** The network below shows the distance, in kilometres, along walkways that connect landmarks A, B, C, D, E, F, G, H and I in a national park.



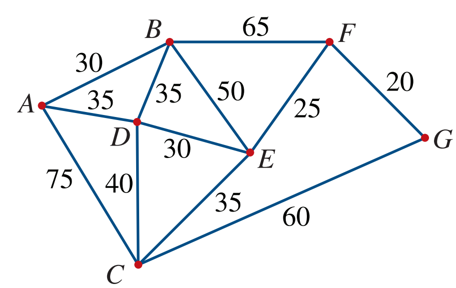
- **a** What distance is travelled on the path $A–B–E–H–I$?
- **b** What distance is travelled on the path $I–G–F–E–D–B–A$?
- **c** What distance is travelled on the circuit $F–E–D–H–E–A–C–F$?
- **d** What distance is travelled on the circuit $D–E–A–B–E–F–G–I–H–D$?
- **e** What is the distance travelled on the shortest cycle starting and finishing at E?
- **f** What is the distance travelled on the shortest cycle starting and finishing at F?
- **g** Find the shortest path and distance travelled from A to I.
- **h** Find the shortest path and distance travelled from C to D.

7 The graph opposite shows a mountain bike rally course. Competitors must pass through each of the checkpoints, A, B, C, D, E and F. The average times (in minutes) taken to ride between the checkpoints are shown on the edges of the graph. Competitors must start and finish at checkpoint A but can pass through the other checkpoints in any order they wish.



- **a** What is the average time travelled on the circuit $A–D–E–F–C–B–A$?
- **b** What is the average time travelled on the circuit $A–B–C–F–E–D–A$?
- **c** What is the average time travelled on the circuit $A–C–E–D–F–C–B–A$?
- **d** Which path would have the shortest average time?
- **e** Will the path with the shortest average time always be the best path? Explain your answer.

8 The network below shows the time (in minutes) of train journeys between seven stations.



- **a** What is the time taken to travel $A–B–E–D–A$?
- **b** What is the time taken to travel $F–G–C–D–B–F$?
- **c** Find the shortest time it would take to travel from A to G.
- **d** Will the path with the shortest time always be the best path? Explain your answer.
- **e** Find the shortest time it would take to travel from A to G if in reality each time the train passes through a station, excluding A and G, an extra 10 minutes is added to the journey.

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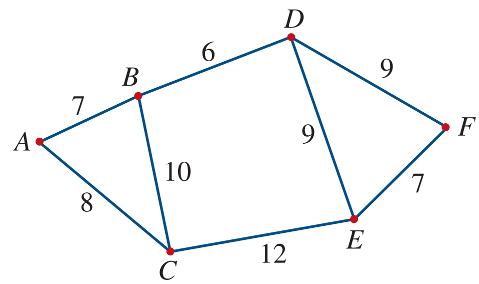
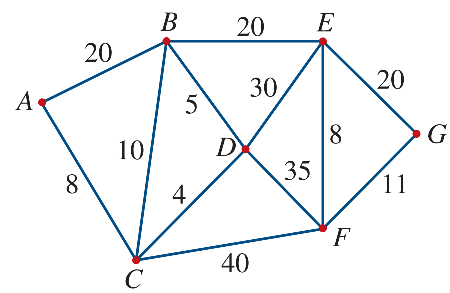
9 The network diagram opposite shows possible water pipes connecting six towns. The numbers on each edge represent the distance (in km) between the towns.



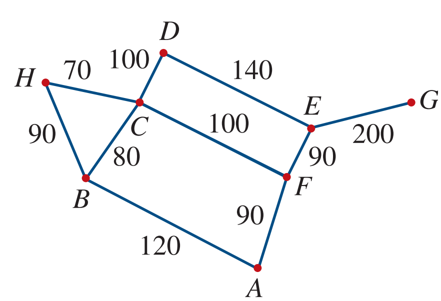
a Which town is closest to *A*? What is the distance?
b Which town is closest to either *A* or your answer in part **a**? What is the distance?
c Which town is closest to either *A* or your answer in part **a** or **b**? What is the distance?
d Which town is closest to either *A* or your answer in part **a** or **b** or **c**? What is the distance?
e Which town is closest to either *A* or your answer in part **a** or **b** or **c** or **d**? What is the distance?
f What is the shortest distance connecting water pipes to the six towns?

10 The network diagram opposite shows possible railway lines connecting seven cities. The numbers on each edge represent the cost (in millions of dollars) in setting up a rail link.



a Which city is closest to *A*? What is the cost?
b Which city is closest to either *A* or your answer in part **a**? What is the cost?
c Which city is closest to either *A* or your answer in part **a** or **b**? What is the cost?
d Which city is closest to either *A* or your answer in part **a** or **b** or **c**? What is the cost?
e Which city is closest to either *A* or your answer in part **a** or **b** or **c** or **d**? What is the cost?
f Which city is closest to either *A* or your answer in part **a** or **b** or **c** or **d** or **e**? What is the cost?
g What is the minimum cost of connecting the seven cities with a railway line?

11 The network diagram opposite shows the major roads connecting eight towns. The numbers on each edge represent the distance in kilometres between the towns.



a Which town is closest to *A*? What is the distance?
b Which town is closest to either *A* or your answer in part **a**? What is the distance?
c Which town is closest to either *A* or your answer in part **a** or **b**? What is the distance?
d Which town is closest to either *A* or your answer in the parts **a** to **c** above? What is the distance?
e Which town is closest to either *A* or your answer in the parts **a** to **d** above? What is the distance?
f Which town is closest to either *A* or your answer in the parts **a** to **e** above? What is the distance?
g Which town is closest to either *A* or your answer in the parts **a** to **f** above? What is the distance?
h What is the minimum distance connecting the eight towns?

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PPT STUDY GUIDE

Key ideas and chapter summary

Network	A network is a term to describe a group or system of interconnected objects. It consists of vertices and edges that indicate a path or route between two objects.	
Network terminology	Network diagram, vertex, edge, degree, loop, directed network, undirected network, weighted edge, walk, trail, path, circuit, cycle, Eulerian trail, Eulerian circuit.	
Drawing a network diagram	Connected graph	A graph is connected if every vertex in the graph is accessible from every other vertex in the graph along a path formed by the edges of the graph.
	Isomorphic graph	Two graphs are isomorphic (equivalent) if: • they have the same numbers of edges and vertices • corresponding vertices have the same degree and the edges connect to the same vertices.
	Weighted graph	A weighted graph is a network diagram that has weighted edges or an edge with a number assigned to it that implies some numerical value such as cost, distance or time.
Network problems	Network diagrams are used to solving problems involving maps and tables.	
Minimum spanning tree	A tree is a connected graph that contains no cycles, multiple edges or loops. A tree with n vertices has $n - 1$ edges. A spanning tree is a tree that connects all of the vertices of a graph. A minimum spanning tree is a spanning tree of minimum length. It connects all the vertices together with the minimum total weighting for the edges. Prim's and Kruskal's algorithms are a set of rules to determine a minimum spanning tree for a graph.	
Shortest path	The shortest path between two vertices in a network is the path where the sum of the weights of its edges is minimised. Problems using the minimal spanning tree to the find least cost to link locations or objects.	

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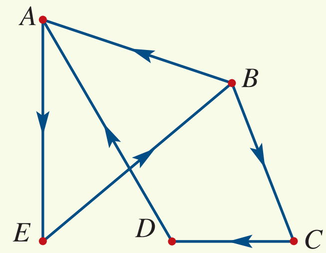
Review

Multiple-choice

Questions **1** to **4** relate to the network diagram opposite.



1 Which of the following vertices has the highest degree?

A Vertex *B*
B Vertex *C*
C Vertex *D*
D Vertex *E*

2 How many edges are in the network diagram?

A 4 **B** 5 **C** 6 **D** 7

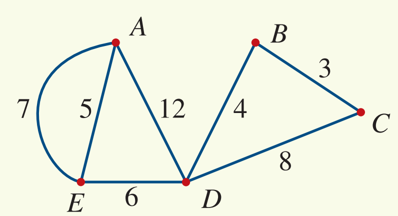
3 Which of the following is a valid path?

A *A–E–B–C* **B** *C–D–A–E–B–A* **C** *C–D–A–E–B–C* **D** *A–E–B*

4 The sequence *C–D–A–E–B* represents:

A a walk only
B a path
C a trail
D a cycle

Questions **5** to **6** relate to the network diagram opposite.



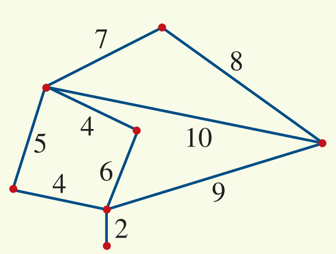
5 Which one of the following is a trail?

A *A–E–D–B–C*
B *E–D–A–B–C–E*
C *A–E–D–C–B–D–A–E*
D *E–D–C–B–D–C–A*

6 What is the length of the shortest path from *A* to *C* in the above network?

A 17 **B** 8 **C** 19 **D** 20

Questions **7** to **8** relate to the network diagram opposite.



7 What is the length of the minimum spanning tree?

A 30
B 31
C 33
D 34

8 What is the length of the shortest path from the bottom vertex to the top vertex?

A 18 **B** 19 **C** 20 **D** 21

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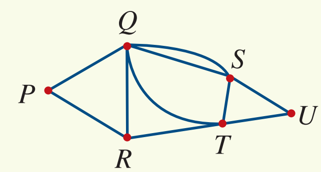
Review

Short-answer

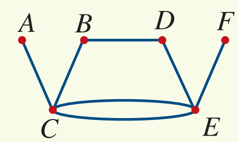
1 Find the degree of the following vertices in the network diagram opposite.

a *P* **b** *Q* **c** *R*
d *S* **e** *T* **f** *U*



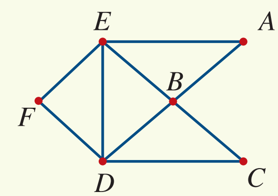
2 A network graph with six vertices is shown opposite.

a What is the degree of each vertex?
b Why does this graph have an Eulerian trail?
c List an Eulerian trail.



3 A network graph with six vertices is shown opposite.

a What is the degree of each vertex?
b Why does this graph have an Eulerian circuit?
c List an Eulerian circuit.

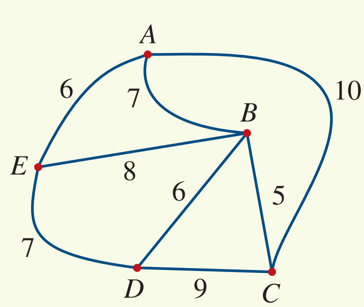


4 A chess tournament is completed between 5 players. Each game has 2 players competing against each other. The table opposite shows the games and the players.

a Draw a network diagram to represent the information in the table.
b What are the vertices of the network diagram?
c Which players have not played a game against each other?

Match	Players	
1	Toby	Jett
2	Amy	Beau
3	Amy	Toby
4	Jett	Ellie
5	Beau	Toby
6	Toby	Ellie

5 For the weighted graph shown, determine the length of the minimum spanning tree.



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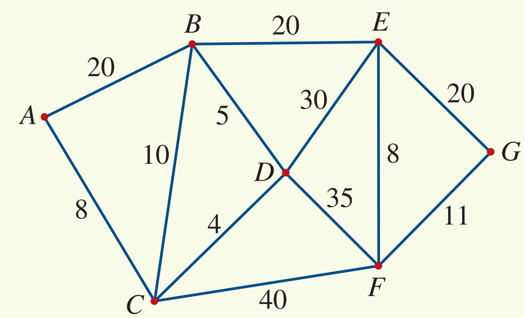
Review

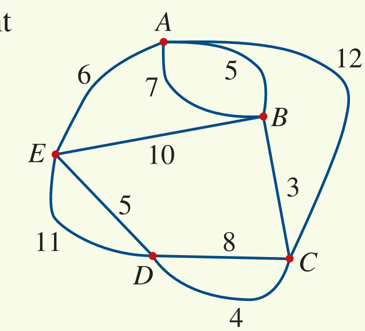
6 What is the length of the minimum spanning tree in the network shown opposite?



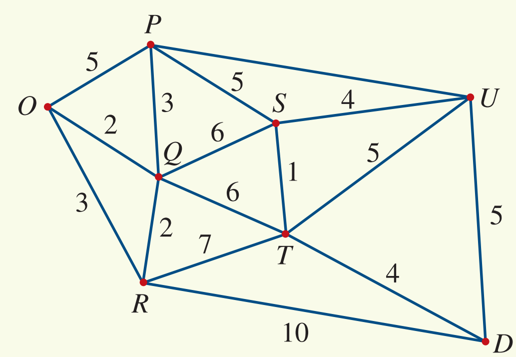
7 In the network shown opposite, the numbers on the edges represent distances in kilometres. Determine the length of:

a the shortest path between vertex A and vertex D

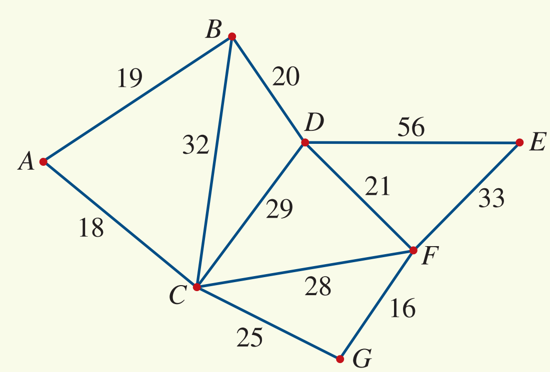
b the length of the minimum spanning tree.



8 What is the length of the shortest path between O and D in the network shown opposite?



9 Seven towns on an island have been surveyed for transport and communications needs. The towns (labelled A, B, C, D, E, F, G) form the network shown here. The road distances between the towns are marked in kilometres. To establish a cable network for communications on the island, it is proposed to put the cable underground beside the existing roads.



a Draw a minimum spanning tree that will ensure that all the towns are connected to the network but that also minimises the amount of cable used.

b What is the minimum length of cable required here if back-up links are not considered necessary; that is, there are no loops in the cable network?

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