# 3 Investments

## Syllabus topic — F2 Investments

This topic will develop your skills to calculate and compare the value of different types of investments over a period of time. In addition, you will gain an understanding of the impact of inflation on prices and wages and the appreciation of items.

## Outcomes

- Calculate simple interest for different rates and periods.
- Compare simple interest graphs for different rates and periods.
- Use the future value formula to calculate the compound interest.
- Solve practical problems involving compound interest.
- Calculate compound interest for different rates and periods.
- Compare compound interest graphs for different rates and periods.
- Compare the growth of simple interest and compound interest.
- Compare and contrast different investment strategies.
- Determine the impact of inflation on prices and wages.
- Calculate the appreciated value of items.

## Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

## Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

## 3A  Simple interest

Interest is the amount paid for borrowing money or the amount earned for lending money. There are different ways of calculating interest. Simple interest (or flat interest) is a fixed percentage of the amount invested or borrowed and is calculated on the original amount. For example, if you invest $100 in a bank account that pays interest at the rate of 5% per annum (per year), you would receive $5 each year. That is,

$$\text{Interest} = \$100 \times \frac{5}{100} = \$5$$

This amount of interest would be paid each year. Simple interest is always calculated on the initial amount, or the principal.

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**SIMPLE INTEREST**

$I = Prn$

$I$ – Interest (simple or flat) earned for the use of money
$P$ – Principal is the initial amount of money borrowed
$r$ – Rate of simple interest per period expressed as a decimal
$n$ – Number of time periods

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### Example 1: Finding simple interest                                    3A

Calculate the amount of simple interest paid on an investment of $16 000 at 8% simple interest per annum for 3 years.

**SOLUTION:**

| | | |
|---|---|---|
| **1** | Write the simple interest formula. | $I = Prn$ |
| **2** | Substitute $P = 16\,000$, $r = 0.08$ and $n = 3$ into the formula. | $= 16\,000 \times 0.08 \times 3$ |
| **3** | Evaluate. | $= \$3840$ |
| **4** | Write the answer in words. | Simple interest is $3840. |

## Amount owed or future value

The interest is added to the principal to determine the amount owed on a loan or the future value of an investment.

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### FORMULA FOR AMOUNT OWED OR FUTURE VALUE

$A = P + I$

$A$ – Amount or final balance
$I$ – Interest (simple or flat) earned
$P$ – Principal is the initial quantity of money

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### Example 2: Calculating the amount owed　　　　　　　　3A

Find the amount owed on a loan of \$50 000 at 7% per annum simple interest at the end of two years and six months.

**SOLUTION:**

| | | |
|---|---|---|
| **1** | Write the simple interest formula. | $I = Prn$ |
| **2** | Substitute $P = 50 000$, $r = 0.07$ and $n = 2.5$ into the formula. | $= 50 000 \times 0.07 \times 2.5$ |
| **3** | Evaluate. | $= \$8750$ |
| **4** | Write the amount owed formula. | $A = P + I$ |
| **5** | Substitute $P = 50 000$ and $I = 8750$ into the formula. | $= 50 000 + 8750$ |
| **6** | Evaluate. | $= \$58 750$ |
| **7** | Write the answer in words. | Amount owed is \$58 750. |

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### Example 3: Calculating value of an investment　　　　　3A

Joel plans to make an investment of \$200 000 at $7\frac{1}{2}$% p.a. simple interest for 2 years. What is the total value of his investment at the end of 2 years?

**SOLUTION:**

| | | |
|---|---|---|
| **1** | Write the simple interest formula. | $I = Prn$ |
| **2** | Substitute $P = 200 000$, $r = 0.075$ and $n = 2$ into the formula. | $= \$200 000 \times 0.075 \times 2$ |
| **3** | Evaluate. | $\$ = \$30 000$ |
| **4** | Write the amount owed formula. | $A = P + I$ |
| **5** | Substitute $P = 50 000$ and $I = 8750$ into the formula. | $= \$200 000 + \$30 000$ |
| **6** | Evaluate. | $= \$230 000$ |
| **7** | Write the answer in words. | Total value is \$230 000. |

## Exercise 3A

**Example 1**   **1**   Calculate the amount of simple interest for each of the following.
  a  Principal = $15 000, Interest rate = 13% p.a., Time period = 3 years
  b  Principal = $2000, Interest rate = $6\frac{1}{2}$% p.a., Time period = 7 years
  c  Principal = $200 000, Interest rate = $9\frac{1}{4}$% p.a., Time period = 2 years
  d  Principal = $3600, Interest rate = 9% p.a., Time period = $3\frac{1}{2}$ years
  e  Principal = $40 000, Interest rate = 7.25% p.a., Time period = $5\frac{1}{4}$ years

**Example 2**   **2**   Calculate the amount owed for each of the following.
  a  Principal = $500, Simple interest rate = 5% p.a., Time period = 4 years
  b  Principal = $900, Simple interest rate = 3% p.a., Time period = 7 years
  c  Principal = $4000, Simple interest rate = $8\frac{1}{2}$ p.a., Time period = 3 years
  d  Principal = $6900, Simple interest rate = 10% p.a., Time period = $4\frac{1}{2}$ years
  e  Principal = $10 000, Simple interest rate = 6.75% p.a., Time period = $2\frac{1}{4}$ years

**3**   The simple interest rate is given as 4.8% per annum.
  a  What is the interest rate per quarter?
  b  What is the interest rate per month?
  c  What is the interest rate per six months?
  d  What is the interest rate per nine months?

**4**   Calculate the amount of simple interest for each of the following.
  a  Principal = $800, Interest rate = 12% p.a., Time period = 1 month
  b  Principal = $1600, Interest rate = 18% p.a., Time period = 6 months
  c  Principal = $60 000, Interest rate = 9.6% p.a., Time period = 3 months
  d  Principal = $20 000, Interest rate = 6% p.a., Time period = 9 months

**5**   Andrew takes a loan of $30 000 for a period of 6 years, at a simple interest rate of 14% per annum. Find the amount owing at the end of 6 years.

**6**   A loan of $1800 is taken out at a simple interest rate of 15.5% per annum. How much interest is owing after 3 months?

**Example 3**   **7**   A sum of $100 000 was invested in a fixed-term account for 4 years. Calculate:
  a  the simple interest earned if the rate of interest is 5.5% per annum
  b  the future value of the investment at the end of 4 years.

**8**   Sophie decides to buy a car for $28 000. She has saved $7000 for the deposit and takes out a loan over 2 years for the balance. The flat rate of interest charged is 12% per annum. What is the total amount of interest to be paid?

9 Domenico has borrowed $24 000 to buy furniture. He wishes to repay the loan over 4 years. Calculate the simple interest on the following rates of interest.
   a  8% per annum for the entire period
   b  9% per annum after a 6-month interest-free period
   c  10% per annum after a 12-month interest-free period

10 Create the spreadsheet below.

**XLS**
**3AQ10**

| SUM | ⇕ | ✕ | ✓ | *fx* | =A5*B5*C5 | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **A** | **B** | **C** | **D** | **E** | **F** | | | |
| 1 | **Mathematics Standard 1** | | | | | | | | |
| 2 | Worksheet to calculate simple interest | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | *Principal* | *Rate* | *Time (yr)* | *Interest* | *Amount* | | | | |
| 5 | $500 | 5.00% | 4.0 | =A5*B5*C5 | $600 | | | | |
| 6 | $15,000 | 13.00% | 3.00 | $5,850 | $20,850 | | | | |
| 7 | $2,000 | 6.25% | 7.00 | $875 | $2,875 | | | | |
| 8 | $200,000 | 9.25% | 2.00 | $37,000 | $237,000 | | | | |
| 9 | $3,600 | 9.00% | 3.50 | $1,134 | $4,734 | | | | |
| 10 | $400,000 | 7.25% | 5.25 | $152,250 | $552,250 | | | | |
| 11 | $800 | 10.00% | 0.50 | $40 | $840 | | | | |
| 12 | $20,000 | 11.50% | 0.75 | $1,725 | $21,725 | | | | |
| 13 | | | | | | | | | |

◄ ►   Simple Interest   +

   a  Cell D5 has a formula that calculates the simple interest. Enter this formula.
   b  The formula for cell E5 is '= A5 + D5'. Fill down the contents of E6 to E12 using this formula.

11 Isabelle buys a TV for $1400. She pays it off monthly over 2 years at an interest rate of 11.5% per annum flat. How much per month will she pay (to the nearest dollar)?

12 Riley wants to earn $4000 a year in interest. How much must he invest if the simple interest rate is 10% p.a.?

13 Samira invests $16 000 for $2\frac{1}{2}$ years. What is the minimum rate of simple interest needed for her to earn $3000?

14 Gurrumul pays back $20 000 on a $15 000 loan at a flat interest rate of 10%. What is the term of the loan?

15 Harry borrowed $300 000 at a flat rate of interest of 8.5% per annum. This rate was fixed for 2 years on the principal. He pays back the interest only over this period.
   a  How much interest is to be paid over the 2 years?
   b  After paying the fixed rate of interest for the first year, Harry finds the bank will decrease the flat interest rate to 7.5% if he pays a charge of $1000. How much will he save by changing to the lower interest rate for the last year?

## 3B  Simple interest graphs

When graphing simple interest make the horizontal axis the time period and the vertical axis the interest earned. Simple interest will increase by a constant amount each time period. This will result in a straight-line graph.

### SIMPLE INTEREST GRAPHS

1. Construct a table of values for $I$ and $n$ using the simple interest formula.
2. Draw a number plane with $n$ the horizontal axis and $I$ the vertical axis. Plot the points.
3. Join the points to make a straight line.

### Example 4: Constructing a simple interest graph                            3B

Draw a graph showing the amount of simple interest earned over a period of 10 years if $1000 is invested at 8% p.a. Use the graph to estimate the interest earned after 7.5 years.

**SOLUTION:**

1. Write the simple interest formula.                $I = Prn$
2. Substitute $P = 1000$, $r = 0.08$ and $n$ into        $= 1000 \times 0.08 \times n$
   the formula.                                      $= 80n$
3. Draw a table of values for $I$ and $n$.
4. Let $n = 0, 2, 4, \ldots$ Find the interest ($I$)
   using $I = 80n$.

| $n$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $I$ | 0 | 160 | 320 | 480 | 640 | 800 |

5. Draw a number plane with $n$ as the horizontal axis and $I$ as the vertical axis.

6. Plot the points $(0, 0)$, $(2, 160)$, $(4, 320)$, $(6, 480)$, $(8, 640)$ and $(10, 800)$.

7. Draw a straight line between the points. Simple interest graphs are linear.

8. Read the graph to estimate $I$ ($I = 600$ when $n = 7.5$).

**Simple interest on $1000 at 8% p.a.**
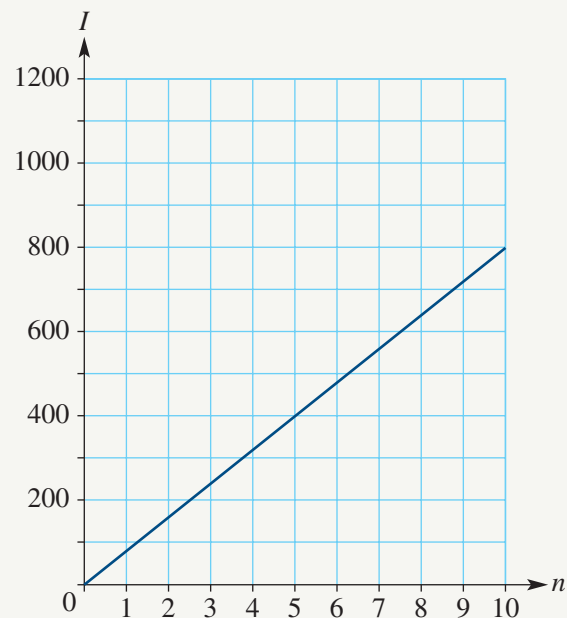


9. Write the answer in words.        Interest after 7.5 years is approximately $600.

## Exercise 3B

**Example 4**  **1**  Aiden invested $1000 at 2% per annum simple interest for 3 years.

    **a**  Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

    **b**  Use this formula to complete the following table of values.

| $n$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $I$ | | | | | |

    **c**  Draw a number plane with $n$ as the horizontal axis and $I$ as the vertical axis.

    **d**  Plot the points from the table of values. Join the points to make a straight line.

**2**  Riley invested $2000 at 6% per annum simple interest for 5 years.

    **a**  Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

    **b**  Use this formula to complete the following table of values.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $I$ | | | | | | |

    **c**  Draw a number plane with $n$ as the horizontal axis and $I$ as the vertical axis.

    **d**  Plot the points from the table of values. Join the points to make a straight line.

    **e**  Use the graph to find the interest after $2\frac{1}{2}$ years.

    **f**  Extend the graph to find the interest after 6 years.

    **g**  Estimate the interest earned after 6 years using the graph.

**3**  Charlotte invested $800 at 7% per annum simple interest for 6 years.

    **a**  Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

    **b**  Use this formula to complete the following table of values.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $I$ | | | | | | | |

    **c**  Draw a number plane with $n$ as the horizontal axis and $I$ as the vertical axis.

    **d**  Plot the points from the table of values. Join the points to make a straight line.

    **e**  Use the graph to find the interest after $2\frac{1}{2}$ years.

    **f**  Extend the graph to find the interest after 7 years.

    **g**  Estimate the interest earned after 7 years using the graph.

**4** Alice is comparing three different interest rates for a possible investment.
  **a** Draw on the same number plane the graph to represent the interest earned over 5 years on:
    **i** $1000 invested at 4% per annum simple interest.
    **ii** $1000 invested at 6% per annum simple interest.
    **iii** $1000 invested at 8% per annum simple interest.
  **b** How much does each investment earn after 5 years?
  **c** Use the graph to estimate the interest earned after $3\frac{1}{2}$ years.
  **d** Find the time for each investment to earn $200 in interest.

**5** Chloe is comparing three different interest rates for a possible investment.
  **a** Draw a graph to represent the interest earned over 5 years on:
    **i** $5000 invested at 5% per annum simple interest
    **ii** $5000 invested at 7% per annum simple interest
    **iii** $5000 invested at 9% per annum simple interest.
  **b** How much does each investment earn after $2\frac{1}{2}$ years?
  **c** How much does each investment earn after 5 years?
  **d** Find the time for each investment to earn $1000 in interest.

**6** Mick is comparing three different interest rates for a possible investment.
  **a** Draw a graph to represent the interest earned for 6 months on:
    **i** $100 000 invested at 6% p.a. simple interest
    **ii** $100 000 invested at 9% p.a. simple interest
    **iii** $100 000 invested at 12% p.a. simple interest
  **b** How much does each investment earn after 1 month?
  **c** How much does each investment earn after 6 months?
  **d** Find the time for each investment to earn $2000 in interest.

**7** The table below gives details for a fixed-term deposit.

| Time period | Simple interest rate per annum |
| --- | --- |
| Less than 3 months | 6.5% |
| 3 to less than 6 months | 7.0% |
| 6 to less than 12 months | 7.5% |
| 12 to less than 24 months | 8.1% |
| 24 to less than 48 months | 8.3% |

Chris has $50 000 to invest in a fixed-term deposit. Draw a separate graph to represent the interest earned on these investments.
  **a** Fixed-term deposit for 3 months
  **b** Fixed-term deposit for 6 months
  **c** Fixed-term deposit for 12 months

## 3C Compound interest – Future value

Compound interest is calculated from the initial amount borrowed or principal plus any interest that has been earned. It calculates interest on the interest. For example, if $100 is invested at a compound interest rate of 10% per annum.

First year –      Interest = $100 × 0.10 × 1 = $10
                     Amount owed = $100 + $10 = $110

Second year –     Interest = $110 × 0.10 × 1 = $11
                     Amount owed = $110 + $11 = $121

Third year –      Interest = $121 × 0.10 × 1 = $12.10
                     Amount owed = $121 + $12.10 = $133.10

These calculations show the interest earned increased each year. In the first year it was $10, the second year $11 and the third year $12.10.

### COMPOUND INTEREST FORMULA – FUTURE VALUE

$FV = PV(1 + r)^n$

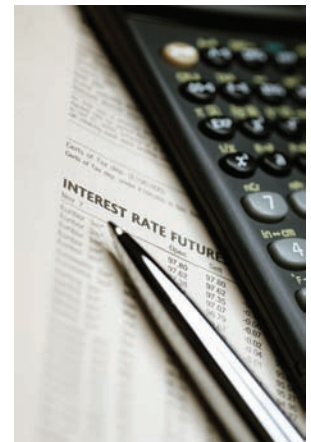$A$ – Amount (final balance) or future value of the loan
$P$ – (initial quantity of money) present value of the loan or principal
$r$ – Rate of interest per compounding time period expressed as a decimal
$n$ – Number of compounding time periods

## Calculating compound interest

The compound interest is calculated by subtracting the principal from the amount borrowed or invested. Alternatively, finance companies provide an investment calculator as an estimate to the value of an investment.

### COMPOUND INTEREST EARNED OR OWED

$I = FV - PV$

$FV$ – future value of the loan or amount (final balance)
$PV$ – Present value of the loan or principal (initial quantity of money)
$I$ – Interest (compound) earned

## Example 5: Finding the compound interest    3C

Paige invests $5000 over 5 years at a compound interest rate of 6.5% p.a. Calculate:

**a**   the amount of the investment after 5 years, correct to the nearest cent

**b**   the interest earned after 5 years, correct to the nearest cent.

**SOLUTION:**

| | | | |
|---|---|---|---|
| **1** | Write the compound interest formula. | **a** | $A = P(1 + r)^n$ |
| **2** | Substitute $P = 5000$, $r = 0.065$ and $n = 5$ into the formula. | | $= 5000(1 + 0.065)^5$ |
| | | | $= 6850.433317$ |
| **3** | Evaluate. | | $= \$6850.43$ |
| **4** | Write the answer in words. | | Amount is $\$6850.43$ |
| | | | |
| **5** | Write the amount borrowed formula. | **b** | $I = A - P$ |
| **6** | Substitute $P = 5000$ and $I = 6850.43$ into the formula. | | $= 6850.43 - 5000$ |
| **7** | Evaluate. | | $= \$1850.43$ |
| **8** | Write the answer in words. | | Interest earned is $\$1850.43$. |

## Example 6: Finding compound interest using a graphics calculator    3C

James borrowed $50 000 for 4 years at 11% p.a. interest compounding monthly.

**a**   What is the amount owed after the 4 years?

**b**   the interest paid after 4 years, correct to the nearest cent.

**SOLUTION:**

| | | | |
|---|---|---|---|
| **1** | Write the compound interest formula. | **a** | $FV = PV(1 + r)^n$ |
| **2** | Substitute $PV = 50000$, $r = \dfrac{0.11}{12}$ and $n = 4 \times 12$ into the formula. | | $= 50000 \times \left(1 + \dfrac{0.11}{12}\right)^{4 \times 12}$ |
| **3** | Evaluate. | | $= 77479.902...$ |
| | | | $\approx \$77479.90$ |
| **4** | Write the answer in words. | | Amount owed is $\$77479.90$. |
| | | | |
| **5** | Write the amount borrowed formula. | **b** | $I = FV - PV$ |
| **6** | Substitute $PV = 50000$ and $FV = 77479.90$ into the formula. | | $= 77479.90 - 50000$ |
| | | | $= \$27479.90$ |
| **7** | Evaluate. | | |
| **8** | Write the answer in words. | | Interest paid is $\$27479.90$. |

## Exercise 3C

**Example 5a**   **1**   Calculate the future value, to the nearest cent, for each of the following.

    **a**   Present value = $400, Compound interest rate = 3% p.a., Time period = 2 years

    **b**   Presentvalue = $26 000, Compound interest rate = 8% p.a., Time period = 4 years

    **c**   Present value = $48 000, Compound interest rate = 3.95% p.a., Time period = 10 years

    **d**   Present value = $3000, Compound interest rate = $5\frac{1}{2}$% p.a., Time period = 5 years

    **e**   Present value = $18 000, Compound interest rate = 10% p.a., Time period = $2\frac{1}{2}$ years

    **f**   Present value = $65 000, Compound interest rate = 5.9% p.a., Time period = $3\frac{1}{4}$ years

    **g**   Present value = $240 000, Compound interest rate = 11.3% p.a., Time period = 4.5 years

    **h**   Present value = $14 000, Compound interest rate = $2\frac{1}{4}$% p.a., Time period = $7\frac{3}{4}$ years

**2**   Use the formula $FV = PV(1 + r)^n$ to calculate the value of an investment of $16 000, over a period of 2 years with an interest rate of 5% compounding annually.

**3**   Tyler sold his car for $35 600. He invested this amount at 7.2% p.a. with interest compounded annually. What is the value of his investment in 15 years?
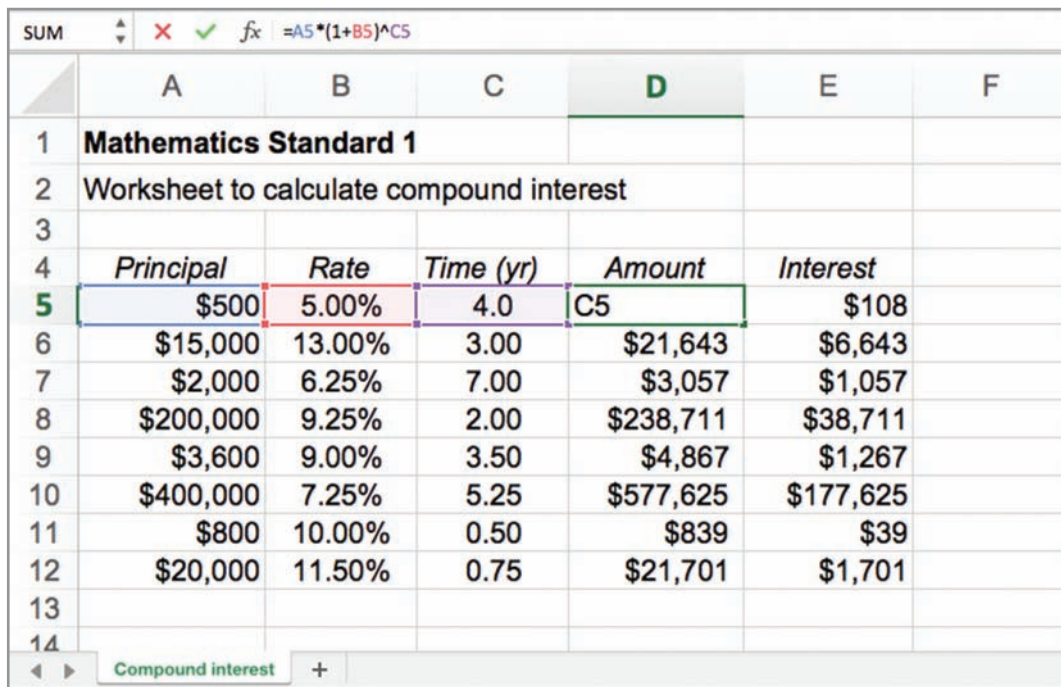
**4**   Sarah wishes to invest $5000 for a period of 8 years. The following investment strategies are suggested to her. What is the interest that will be earned on each investment strategy? Answer to the nearest dollar.

    **a**   Simple interest at 7% p.a.

    **b**   Compound interest at 7% p.a. compounded annually

    **c**   Simple interest at 14% p.a.

    **d**   Compound interest at 14% p.a. compounded annually

**Example 5b**   **5**   Calculate the amount of compound interest for each of the following.

    **a**   Futurevalue = $25 000, Interest rate = 7% p.a., Time period = 5 years

    **b**   Future value = $300 000, Interest rate = $10\frac{1}{4}$% p.a., Time period = 3 years

    **c**   Future value = $6500, Interest rate = 13% p.a., Time period = $1\frac{1}{2}$ years

    **d**   Future value = $80 000, Interest rate = 8.25% p.a., Time period = $3\frac{1}{4}$ years

**6** Alexis and Oscar invested $27 000 for 6 years at 9% p.a. interest compounding annually. What is the amount of interest earned? Answer to the nearest cent.

**7** What is the interest earned if $15 720 is invested for 5 years, compounding yearly at 8% p.a.? Answer to the nearest dollar.

**8** Find the amount $7800 will grow to if it is compounded at 6% p.a. for:

  **a** 3 years          **b** 10 years          **c** 30 years.

**9** Create the spreadsheet below.

XLS
3CQ9

| SUM | ▲▼ | ✕ | ✓ | fx | =A5*(1+B5)^C5 | | |
|---|---|---|---|---|---|---|---|
| | **A** | **B** | **C** | **D** | **E** | **F** | |
| 1 | **Mathematics Standard 1** | | | | | | |
| 2 | Worksheet to calculate compound interest | | | | | | |
| 3 | | | | | | | |
| 4 | *Principal* | *Rate* | *Time (yr)* | *Amount* | *Interest* | | |
| 5 | $500 | 5.00% | 4.0 | C5 | $108 | | |
| 6 | $15,000 | 13.00% | 3.00 | $21,643 | $6,643 | | |
| 7 | $2,000 | 6.25% | 7.00 | $3,057 | $1,057 | | |
| 8 | $200,000 | 9.25% | 2.00 | $238,711 | $38,711 | | |
| 9 | $3,600 | 9.00% | 3.50 | $4,867 | $1,267 | | |
| 10 | $400,000 | 7.25% | 5.25 | $577,625 | $177,625 | | |
| 11 | $800 | 10.00% | 0.50 | $839 | $39 | | |
| 12 | $20,000 | 11.50% | 0.75 | $21,701 | $1,701 | | |
| 13 | | | | | | | |
| 14 | | | | | | | |
| ◄ ► | Compound interest | + | | | | | |

  **a** Cell D5 has a formula that calculates the compound interest. Enter this formula.
  **b** The formula for cell E5 is '= A5 + D5'. Fill down the contents of E6 to E12 using this formula.

**10** Jack has $50 000 to invest for two years. Which is the better investment? Why?

| **Investment 1** Simple interest rate 4% p.a. | **Investment 2** Compound interest rate 4% p.a. |
|---|---|

**Example 6** **11** Calculate the future value, to the nearest cent, for each of the following.

    **a** Present value of $680 invested for 4 years at 5% p.a. compounded biannually

    **b** Present value of $1250 invested for 8 years at 3% p.a. compounded biannually

    **c** Present value of $5000 invested for 6 years at 6% p.a. compounded quarterly

    **d** Present value of $23 000 invested for 5 years at 7% p.a. compounded quarterly

    **e** Present value of $1400 invested for 3 years at 4.2% p.a. compounded monthly

    **f** Present value of $4680 invested for 10 years at 8% p.a. compounded monthly

    **g** Present value of $780 invested for 5 years at 9.8% p.a. compounded weekly

    **h** Present value of $1340 invested for 6 years at 6% p.a. compounded weekly

    **i** Present value of $290 invested for 7 years at 10% p.a. compounded fortnightly

**12** Which of the following is the best investment over 25 years? Justify your answer.

    Investment A: Simple interest at 4% p.a. with $100 000

    Investment B: Compound interest at 4% p.a. compounded annually with $100 000

    Investment C: Compound interest at 4% p.a. compounded biannually with $100 000

    Investment D: Compound interest at 4% p.a. compounded quarterly with $100 000

    Investment E: Compound interest at 4% p.a. compounded monthly with $100 000

**13** Find the future value in a bank account after 3 years if the present value of $4000 earns 4.6% p.a. compound interest, paid quarterly.

**14** Jackson invested $16 400 over 6 years at 7.4% p.a. interest compounding monthly. Calculate the:

    **a** value of the investment after 4 years

    **b** compound interest earned.

**15** Use the formula $A = P(1 + r)^n$ to calculate the value of an investment of $10 000, over a period of 2 years with a monthly interest rate of 0.8% compounding monthly.

**16** Sebastian invested $20 000 at 12% p.a. interest compounding monthly. What is the amount of interest earned in the first year?

**17** Find the amount of money in a bank account after 6 years if an initial amount of $4000 earns 8% p.a. compound interest, paid quarterly.

**18** Isabella invested $13 500 over 7 years at 6.2% p.a. interest compounding quarterly. Calculate the:

    **a** value of the investment after 7 years (to the nearest cent)

    **b** compound interest earned (to the nearest cent).

## 3D Compound interest – Present value

The compound interest formula to find the future value can be rearranged with the present value as the subject of the formula. This process is shown here.

$$FV = PV(1 + r)^n$$

$$\frac{FV}{(1 + r)^n} = \frac{PV(1 + r)^n}{(1 + r)^n}$$

$$\frac{FV}{(1 + r)^n} = PV$$

$$PV = \frac{FV}{(1 + r)^n}$$

### COMPOUND INTEREST FORMULA – PRESENT VALUE

$$PV = \frac{FV}{(1 + r)^n}$$

$FV$ – Future value of the loan or amount (final balance)

$PV$ – Present value of the loan or principal (initial quantity of money)

$r$ – Rate of interest per compounding time period expressed as a decimal

$n$ – Number of compounding time periods

### Example 7: Calculating the present value     3D

**a** Calculate the present value of an annuity whose future value is $8723.27 over 5 years at a compound interest rate of 4.5% p.a. Answer correct to the nearest dollar.

**b** Calculate the present value of an annuity whose future value is $500 000 over 8 years with an interest rate of 8.5% per annum compounded monthly. Answer correct to the nearest cent.

**SOLUTION:**

**1** Write the present value formula.

**a** $PV = \dfrac{FV}{(1 + r)^n}$

**2** Substitute $FV = \$8723.27$, $r = 0.045$ and $n = 5$ into the formula.

$= \dfrac{8723.27}{(1 + 0.045)^5}$

**3** Evaluate to the nearest cent.

$= 6999.997..$

$\approx \$7000$

**4** Write the answer in words.

$\therefore$ Present value is $7000

**5** Write the present value formula.

**b** $PV = \dfrac{FV}{(1 + r)^n}$

**6** The investment is compounding per month hence the rate ($r$) and time period ($n$) are expressed in months.

**7** Substitute $FV = 500 000$, $r = \dfrac{0.085}{12}$ and $n = 8 \times 12 = 96$.

$= \dfrac{500 000}{\left(1 + \dfrac{0.085}{12}\right)^{96}}$

**8** Evaluate to the nearest cent.

$= \$253 916.41$

**9** Write the answer in words.

Present value is $253 916.41.

## Exercise 3D

**Example 7a**   **1**   Calculate the present value, to the nearest cent, for each of the following.

    **a**   Future value = $34 000, Interest rate = 4% p.a., Time period = 4 years

    **b**   Future value = $87 000, Interest rate = 5% p.a., Time period = 12 years

    **c**   Future value = $190 000, Interest rate = 3% p.a., Time period = 15 years

    **d**   Future value = $200 000, Interest rate = $12\frac{1}{4}$% p.a., Time period = 5 years

    **e**   Future value = $4600, Interest rate = 15% p.a., Time period = $2\frac{1}{2}$ years

    **f**   Future value = $60 000, Interest rate = 6.25% p.a., Time period = $1\frac{1}{4}$ years

    **g**   Future value = $320 000, Interest rate = 5.5% p.a., Time period = $9\frac{3}{4}$ years

    **h**   Future value = $450 000, Interest rate = $9\frac{1}{2}$% p.a., Time period = 25 years

**2**   What sum of money would Zoe need to invest to accumulate a total of $50 000 at the end of 4 years at 6% p.a. compound interest? Answer to the nearest cent.

**3**   Calculate the amount that must be invested at 9.3% p.a. interest compounding annually to have $70 000 at the end of 3 years. Answer to the nearest cent.

**4**   What sum of money needs to be invested to accumulate to a total of $100 000 in 10 years at 7.25% p.a. compound interest? Answer to the nearest cent.

**5**   Find the present value of money in a bank account if the future value after four years earning 9% p.a. compound interest, paid annually, is $5000. Answer to the nearest dollar.

**Example 7b**   **6**   Calculate the present value, to the nearest dollar, for each of the following.

    **a**   Future value of $1243, interest rate at 6% p.a. compounded biannually for 5 years

    **b**   Future value of $8200, interest rate at 4% p.a. compounded quarterly for 8 years

    **c**   Future value of $1580, interest rate at 4.8% p.a. compounded monthly for 4 years

    **d**   Future value of $19 600, interest rate at 8% p.a. compounded weekly for 3 years

    **e**   Future value of $3800, interest rate at 5% p.a. compounded fortnightly for 7 years

**7**   What sum of money would Levi need to invest to accumulate a total of $100 000 at the end of 7 years at 8% p.a. interest compounding biannually? Answer to the nearest cent.

**8**   What sum of money needs to be invested to accumulate to a total of $40 000 in 10 years at 9.25% p.a. interest compounding monthly? Answer to the nearest cent.

## 3E Compound interest graphs

When graphing compound interest make the horizontal axis the compounding time periods ($n$) and the vertical axis the interest earned ($I$). Compound interest will increase by a different amount each time period. This will result in an exponential curve.

> **COMPOUND INTEREST GRAPHS**
>
> 1 Construct a table of values for $I$ and $n$ using the compound interest formula.
> 2 Draw a number plane with $n$ the horizontal axis and $I$ the vertical axis. Plot the points.
> 3 Join the points to make an exponential curve.

### Example 8: Constructing a compound interest graph 　　　　　　　　3E

Draw a graph showing the interest earned over a period of 10 years if $1000 is invested at a compound interest rate of 8% p.a. Use the graph to estimate the interest earned after 7.5 years.

**SOLUTION:**

1 Write the future value and interest earned formulas.

2 Substitute $PV = 1000$, $r = 0.08$ and $n$ into the formula.

$$FV = PV(1 + r)^n \qquad\qquad I = FV - PV$$
$$= 1000 \times (1.08)^n \qquad = PV(1 + r)^n - PV$$
$$= 1000(1.08)^n - 1000$$

3 Draw a table of values for $n$, $FV$ and $I$

4 Let $n = 0, 2, 4, \ldots$ Find the future value and interest earned.

| $n$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $FV$ | 1000 | 1166 | 1360 | 1587 | 1851 | 2159 |
| $I$ | 0 | 166 | 360 | 587 | 851 | 1159 |

5 Draw a number plane with $n$ as the horizontal axis and $I$ as the vertical axis.

6 Plot the points (0, 0), (2, 166), (4, 360), (6, 587), (8, 851) and (10, 1159).

7 Draw an exponential curve (not a straight line) between the points.

8 Read the graph to estimate $I$ when $n = 7.5$ years ($I = $780 when $n = 7.5$).

Compound interest earned on $1000 at 8% p.a.
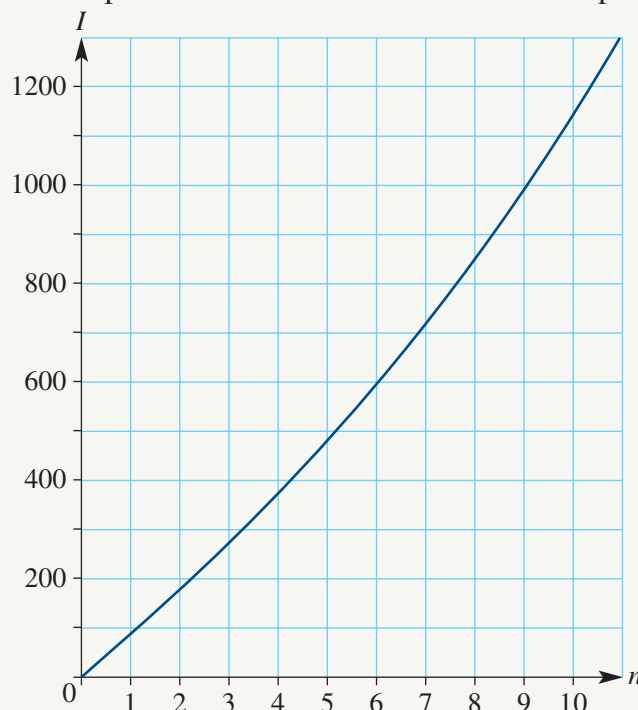


9 Write the answer in words. 　　Interest on the loan after 7.5 years is about $780.

## Exercise 3E

**Example 8** **1** Chloe invested $2000 at 6% per annum interest compounding annually for 5 years.

    **a** Substitute the present value and the interest rate into the formula $FV = PV(1 + r)^n$ to obtain an expression for the future value.

    **b** Substitute the future value expression and the present value into the formula $I = FV - PV$.

    **c** Use these formulas to complete the following table of values. Answer to nearest dollar.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|---|---|---|---|
| $FV$ | | | | | | |
| $I$ | | | | | | |

    **d** Draw a number plane with $n$ as the horizontal axis and $I$ as the vertical axis.

    **e** Plot the points from the table of values. Join the points to make a curve.

    **f** Extend the graph to find the interest after 6 years.

**2** Samuel invested $800 at 7% p.a. compound interest, paid annually, for 6 years.

    **a** Substitute the present value and the interest rate into the formula $FV = PV(1 + r)^n$ to obtain an expression for the future value.

    **b** Substitute the future value expression and the present value into the formula $I = FV - PV$.

    **c** Use these formulas to complete the following table of values. Answer to nearest dollar.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|-----|---|---|---|---|---|---|---|
| $FV$ | | | | | | | |
| $I$ | | | | | | | |

    **d** Draw a number plane with $n$ as the horizontal axis and $I$ as the vertical axis.

    **e** Plot the points from the table of values. Join the points to make a curve.

    **f** Use the graph to find the interest after $2\frac{1}{2}$ years.

    **g** Extend the graph to find the interest after 7 years.

**3** Mitchell is comparing three different interest rates for a possible investment.

    **a** Draw on the same number plane the graph to represent the interest earned over 5 years on:

        **i** $1000 invested at 4% per annum interest compounding annually

        **ii** $1000 invested at 6% per annum interest compounding annually

        **iii** $1000 invested at 8% per annum interest compounding annually.

    **b** What is the approximate value of the interest on each investment after 5 years?

    **c** What is the approximate value of the interest on each investment after $3\frac{1}{2}$ years?

    **d** Find the approximate time for each investment to earn $200 in interest.

**4** Draw a graph showing the amount of the loan over a period of 6 years if $1000 is borrowed at a compound interest rate of 10% p.a. Use the graph to estimate the interest after $5\frac{1}{2}$ years.

**5** Henry is comparing three different interest rates for a possible investment.
  **a** Draw a graph to represent the interest earned for 1 year on:
   **i** $5000 invested at 4% p.a. interest compounding quarterly
   **ii** $5000 invested at 8% p.a. interest compounding quarterly
   **iii** $5000 invested at 12% p.a. interest compounding quarterly.
  **b** How much does each investment earn after 1 quarter?
  **c** How much does each investment earn after 3 quarters?
  **d** Find the time for each investment to earn $200 in interest.

**6** Ruby is comparing three different interest rates for a possible investment.
  **a** Draw a graph to represent the interest earned over 6 months on:
   **i** $100000 invested at 6% p.a. interest compounding monthly
   **ii** $100000 invested at 9% p.a. interest compounding monthly
   **iii** $100000 invested at 12% p.a. interest compounding monthly.
  **b** What is the approximate value of interest earned on each investment after 2 months?
  **c** How much does each investment earn after 6 months?
  **d** Find the time for each investment to earn $3000 in interest.

**7** The table below gives details for an investment product. The compound interest earned is paid quarterly.

| Investment | Rate of compound interest |
|------------|---------------------------|
| A | 4% p.a. |
| B | 6% p.a. |
| C | 8% p.a. |
| D | 10% p.a. |

Ethan is prepared to invest $50000 in the above product.
  **a** Draw a graph to represent the interest earned on these investments after 3 years.
  **b** What is the interest earned on investment B after 2 years?
  **c** What is the interest earned on investment C after 18 months?
  **d** Find the approximate time for investment D to earn $10000 in interest.

# 3F Appreciation and inflation

## Appreciation

Appreciation is the increase in value of items such as art, gold or land. This increase in value is often expressed as the rate of appreciation. Calculating the appreciation is similar to calculating the compound interest. For example, a painting worth $100 000 that has an annual rate of appreciation of 10% will be worth $110 000 after one year (an increase of $10 000). In the second year its value will increase by $11 000. The amount of appreciation has increased.

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### APPRECIATION

$FV = PV(1 + r)^n$ or $A = P(1 + r)^n$

$FV$ – Future value of the item

$PV$ – Present value of the item

$r$ – Rate of appreciation per compounding time period expressed as a decimal

$n$ – Number of compounding time periods

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### Example 9: Finding the appreciated value                3F

Joel bought a unit for $690 000. If the unit appreciates at 9% p.a., what is its value after 7 years? Answer to the nearest dollar.

**SOLUTION:**

1  Write the formula for appreciation $FV = PV(1 + r)^n$.

2  Substitute $PV = \$690\,000$, $r = 0.09$ (9% expressed as a decimal) and $n = 7$ into the formula.

3  Evaluate.

4  Write the answer to the correct degree of accuracy.

5  Answer the question in words.

$$FV = PV(1 + r)^n$$
$$= 690\,000(1 + 0.09)^7$$

$$= 1\,261\,346.993$$
$$\approx \$1\,261\,347$$

Unit is valued at $1 261 347.

## Inflation

Inflation is a rise in the price of goods and services or Consumer Price Index (CPI). It is measured by comparing the prices of a fixed basket of goods and services. If inflation rises then a person's spending power decreases. The inflation rate is the annual percentage change in the CPI. In Australia, the Reserve Bank aims to keep the inflation rate in a 2% to 3% band.

Calculating inflation is similar to calculating appreciation or compound interest.

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**INFLATION**

Inflation rate is the annual percentage change in the CPI.

Use the formula $FV = PV(1 + r)^n$ to calculate the future value of an item following inflation.

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**Example 10: Finding the price of goods following inflation**                                **3F**

**a** What is the price of a $650 clothes dryer after one year following inflation? (Inflation rate is 2.6% p.a.)

**b** What is the price of a $400 clothes dryer after three years following inflation? (Inflation rate is 3.2% p.a.)

**SOLUTION:**

**1** Write the formula for inflation.

**2** Substitute $PV = 650$, $r = 0.026$ and $n = 1$ into the formula.

**3** Evaluate correct to two decimal places.

**4** Write the answer in words.

**a** $FV = PV(1 + r)^n$
$= 650(1 + 0.026)^1$
$= \$666.90$

Clothes dryer will cost $666.90.

**1** Write the formula for inflation.

**2** Substitute $PV = 400$, $r = 0.032$ and $n = 3$ into the formula.

**3** Evaluate correct to two decimal places.

**4** Write the answer in words.

**b** $FV = PV(1 + r)^n$
$= 400(1 + 0.032)^3$
$= \$439.64$

Clothes dryer will cost $439.64.

## Exercise 3F

Example 9   **1**   A vintage car was bought for $70 000 and appreciated at the rate of 6% p.a. What will be the value of the car after 4 years? Answer correct to the nearest cent.

**2**   The price of a house has increased by 4.5% for each of the last two years. It was bought for $490 000 two years ago. What is the new current value?

**3**   William bought the following antiques.

   **a**   Tall boy valued at $4450. Each year its value appreciated by 5%. Calculate the value of the tall boy after 3 years. Answer correct to the nearest cent.

   **b**   Table valued at $6200. Each year its value appreciated by 4%. Calculate the value of the table after 5 years. Answer correct to the nearest cent.

   **c**   Chair valued at $1250. Each year its value appreciated by 9%. Calculate the value of the chair after 4 years. Answer correct to the nearest cent.

**4**   The collection of dolls was valued at $1500 four years ago. If it appreciated at 12% p.a., find its current value. Answer correct to the nearest cent.

**5**   The price of a diamond ring has increased from $3400 to $5300 during the past five years due to inflation. What is the rise in the price of the ring?

**6**   Patrick bought an apartment for $740 000. If the apartment appreciates at 8% p.a., what is its value after 6 years? Answer to the nearest dollar.

**7**   A retailer advertises an antique bed for $1250. If the bed appreciates at 5% p.a., what is its value after 4 years? Answer to the nearest dollar.

**8**   Scarlett bought a painting for $240. If the painting appreciates at 11% p.a., what is its value after 7 years? Answer to the nearest dollar.

**9**   Gold was bought for $4800. It appreciates by 2% p.a. Find the value of the gold after 3.5 years. Answer to the nearest dollar.

Example 10  **10** What is the price of a $1900 television after one year following inflation? (Inflation rate is 2.86% p.a.) Answer to the nearest dollar.

**11** What is the price of a $500 lawn mower after 3 years following inflation? (Inflation rate is 3.5% p.a.) Answer to the nearest dollar.

**12** What is the price of a $390 printer after 4 years following inflation? (Inflation rate is 5.2% p.a.) Answer to the nearest dollar.

**13** The average inflation for the next 5 years is predicted to be 3%. Calculate the price of the following goods in 5 years time. Answer correct to the nearest cent.
   **a** 3 L of milk for $3.57
   **b** Loaf of bread for $3.30
   **c** 250 g honey for $4.50
   **d** 800 g of eggs for $5.20

**14** If the inflation rate is 5% p.a., what would you expect to pay, to the nearest dollar, in 4 years time for a house that costs:
   **a** $280 000?                        **b** $760 000?
   **c** $324 000?                        **d** $580 000?
   **e** $1 260 000?                      **f** $956 000?

**15** If the inflation rate is 5% p.a., what would you expect to pay, to the nearest dollar, in 4 years time for a new motor vehicle that costs:
   **a** $34 000?                         **b** $22 500?
   **c** $65 000?                         **d** $19 990?
   **e** $120 000?                        **f** $57 200?

**16** The graph below shows the value of cricket memorabilia for the past 6 years.
   **a** What was the value of the memorabilia after 4 years?
   **b** What was the value of the memorabilia after 6 years?
   **c** What was the initial value?
   **d** How much did the memorabilia appreciate each year?

**Cricket memorabilia**
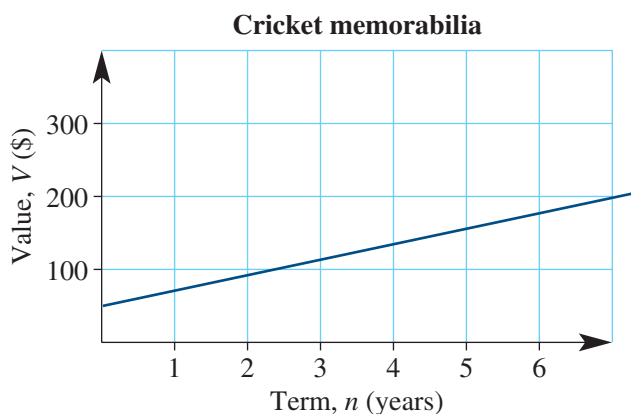
# Key ideas and chapter summary

**Simple interest**
$$I = Prn \qquad A = P + I$$
$I$ – Interest (simple or flat) earned for the use of money
$P$ – Principal is the initial amount of money borrowed
$r$ – Rate of simple interest per period expressed as a decimal
$n$ – Number of time periods
$A$ – Amount or final balance

**Simple interest graphs**
1 Construct a table of values for $I$ and $n$ using $I = Prn$.
2 Draw a number plane – $n$ is the horizontal axis, $I$ is the vertical axis
3 Plot the points and join them to make a straight line.

**Compound interest future value**
$$FV = PV(1 + r)^n \text{ or } I = FV - PV$$
$FV$ – Future value of the loan or amount (final balance)
$PV$ – Present value of the loan or principal (initial quantity of money)
$r$ – Rate of interest per compounding time period as a decimal
$n$ – Number of compounding time periods
$I$ – Interest (compounded) earned

**Compound interest present value**
$$PV = \frac{FV}{(1 + r)^n}$$
$FV$ – Future value of the loan or amount (final balance)
$PV$ – Present value of the loan or principal (initial quantity of money)
$r$ – Rate of interest per compounding time period as a decimal
$n$ – Number of compounding time periods

**Compound interest graphs**
1 Construct a table of values for $n$ and $I$ using $FV = PV(1 + r)^n$ and $I = FV - PV$.
2 Draw a number plane – $n$ is the horizontal axis, $I$ is the vertical axis
3 Plot the points and join them to make a curve.

**Appreciation and inflation**
Use the formula $FV = PV(1 + r)^n$ for appreciation and inflation.
Inflation rate is the annual percentage change in the CPI.

**Review**

## Multiple-choice

**1** What is the flat-rate interest on $1400 at 7% p.a. for 3 years?

    **A** $98          **B** $196          **C** $294          **D** $498

**2** Gemma invested $800 for 2 years at a simple interest rate of 4% per annum. What is the total amount of interest earned by the investment?

    **A** $32          **B** $64          **C** $160          **D** $320

**3** Lincoln wants to earn $9000 a year in interest. What must he invest if the simple interest rate is 15% p.a.? Answer to the nearest dollar.

    **A** $1350          **B** $10350          **C** $60000          **D** $600000.

**4** Using the graph, what is the interest after $3\frac{1}{2}$ years?

    **A** $120          **B** $130

    **C** $140          **D** $240

**Simple interest at 4% p.a.**

**5** What was the amount of the investment shown in the graph?

    **A** $1000          **B** $2000

    **C** $3000          **D** $4000

**6** Alana invests $8000 at 10% p.a. interest compounding annually. What is the future value after 3 years? (Answer to the nearest dollar.)

    **A** $242          **B** $2648          **C** $8242          **D** $10648

**7** George borrows $3000 at 10% p.a. interest compounding annually. What is the interest earned after 2 years? (Answer to the nearest dollar.)

    **A** $630          **B** $1500          **C** $6000          **D** $3630

**8** The compound interest on $4600 at 12% p.a. for 2 years is:

    **A** $1104          **B** $1170          **C** $4600          **D** $5700

**9** A painting was bought for $460000 and appreciated at the rate of 7% p.a. What will be the value of the painting after 4 years? (Answer to the nearest dollar.)

    **A** $473016          **B** $492200          **C** $588800          **D** $602966
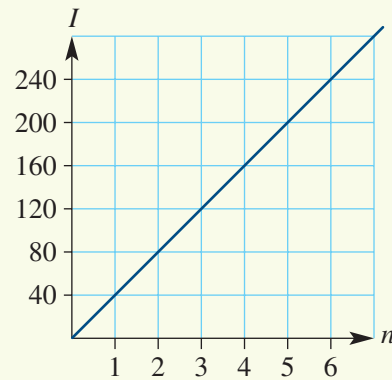
## Short-answer

**1** Charles takes out a flat-rate loan of $60 000 for a period of 5 years, at a simple interest rate of 12% per annum. Find the amount owing at the end of 5 years.

**2** Keira would like to purchase a $2000 TV from an electronics shop. However, to buy the TV, she has applied for a flat-rate loan over 2 years at 15% p.a. How much does Amelia pay altogether for the TV?

**3** Nate borrowed $1800 at 6% per annum. What is the simple interest accrued between 30 June and 1 September?

**4** Kayla borrowed $36 000 at a flat rate of interest of 7% per annum for $3\frac{1}{2}$ years. How much interest did she pay? Answer to the nearest dollar.

**5** Sam invested $1000 at 7% per annum simple interest for 4 years.
  **a** Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.
  **b** Use this formula to complete the following table of values.

| $n$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $I$ | | | | | |

  **c** Draw a number plane with $n$ as the horizontal axis and $I$ as the vertical axis.
  **d** Plot the points from the table of values. Join the points to make a straight line.
  **e** Use the graph to find the interest after $2\frac{1}{2}$ years.
  **f** Extend the graph to find the interest after 6 years.
  **g** Find the time when the interest is $210.

**6** Caitlin invested $1000 at 5% per annum simple interest for 6 years.
  **a** Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.
  **b** Use this formula to complete the following table of values.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $I$ | | | | | | |

  **c** Draw a number plane with $n$ as the horizontal axis and $I$ as the vertical axis.
  **d** Plot the points from the table of values. Join the points to make a straight line.
  **e** What is the interest after $5\frac{1}{2}$ years?

**7** Calculate the future value, to the nearest cent, for each of the following.

  **a** Present value = $920, Compound interest rate = 5% p.a., Time period = 4 years

  **b** Present value of $2100 invested for 3 years at 6.1% p.a. compounded monthly.

**8** Calculate the present value, to the nearest cent, for each of the following.

  **a** Future value = $26000, Interest rate = 4.9% p.a., Time period = 3 years

  **b** Future value of $10400, Interest rate at 9% p.a. compounded quarterly for 5 years.

**9** What sum of money would Emma need to invest to accumulate a total of $200000 at the end of 10 years at 12% p.a. interest compounding biannually? Answer to the nearest cent.

**10** Declan invested $1600 at 10% p.a. compound interest, paid annually, for 6 years.

  **a** Substitute the present value and the interest rate into the formula $FV = PV(1 + r)^n$ to obtain an expression for the future value.

  **b** Substitute the future value expression and the present value into the formula $I = FV - PV$.

  **c** Use these formulas to complete the following table of values. Answer to nearest dollar.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|-----|---|---|---|---|---|---|---|
| $FV$ | | | | | | | |
| $I$ | | | | | | | |

  **d** Draw a number plane with $n$ as the horizontal axis and $I$ as the vertical axis.

  **e** Plot the points from the table of values. Join the points to make a curve.

  **f** Use the graph to find the interest after $1\frac{1}{2}$ years.

**11** An investment is appreciating at a rate of 4% of its value each year. Ruby decides to invest $480000.

  **a** What will be the investment's value after 10 years? Answer to the nearest dollar.

  **b** How much does the investment increase during the first 10 years?

**12** The average inflation for the next five years is predicted to be 2.5%. Calculate the price of the following goods in 3 years time. Answer to the nearest cent.

  **a** 2L of soft drink for $2.80

  **b** Apple pie for $4.60

  **c** Hamburger for $6.00

  **d** Bottle of water for $1.60

  **e** Punnet of strawberries for $4.50

  **f** 500g of chicken breast for $8.90