Practice set 3

For Questions **1** to **11**, select the correct answer **A**, **B**, **C** or **D**.

1 The quotient rule for differentiating $y = \frac{u}{v}$ is:

A $y' = \frac{uv' - vu'}{v^2}$ **B** $\frac{u'v - v'u}{v^2}$

C $y' = u'v + v'u$ **D** $y' = uv' + vu'$

2 If $f(x) = x^2$ and $g(x) = 2x + 1$, the composite function $g(f(x))$ is given by:

A $(2x + 1)^2$ **B** $(2x)^2 + 1$

C $2x + 1^2$ **D** $2x^2 + 1$

3 The number of employees, N, is inversely proportional to the time, t, it takes to do a stocktake. What is the equation showing this information?

A $N = kt$ **B** $N = t + k$ **C** $N = \frac{k}{t}$ **D** $N = \frac{t}{k}$

4 Find the derivative of $(3x - 2)^8$.

A $(3x - 2)^7$ **B** $8(3x - 2)^7$

C $8x^7(3x - 2)$ **D** $24(3x - 2)^7$

5 Find the probability of drawing out a blue and a white ball from a bag containing 7 blue and 5 white balls if the first ball is not replaced before taking out the second.

A $\frac{70}{121}$ **B** $\frac{70}{144}$ **C** $\frac{1225}{17\,424}$ **D** $\frac{35}{66}$

6 The equation of a circle with radius 3 and centre (–1, 4) is:

A $(x - 1)^2 + (y + 4)^2 = 3$ **B** $(x - 1)^2 + (y + 4)^2 = 9$

C $(x + 1)^2 + (y - 4)^2 = 9$ **D** $(x + 1)^2 + (y - 4)^2 = 3$

7 If $f(x) = 2x^2 - 3x + 1$ and $g(x) = (x + 3)^2$ find the degree of $y = f(x)g(x)$.

A 2 **B** 4 **C** 3 **D** 5

8 Find the domain of $f(x) = \frac{2}{x + 7}$.

A $(-\infty, 7) \cup (7, \infty)$ **B** $(-\infty, -7) \cup (-7, \infty)$

C $(-\infty, 7) \cap (7, \infty)$ **D** $(-\infty, -7) \cap (-7, \infty)$

9 If the displacement of a particle is given by $x = 2t^3 + 6t^2 - 4t + 10$, the initial velocity is:

A –4 **B** 10 **C** 12 **D** 14

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10 In a group of 25 students, 19 catch a train to school and 21 catch a bus. If one of these students is chosen at random, find the probability that the student only catches a bus to school.

A $\frac{6}{25}$ **B** $\frac{21}{25}$ **C** $\frac{3}{5}$ **D** $\frac{3}{20}$

11 Conditional probability $P(A|B)$ is given by:

A $\frac{P(A \cup B)}{P(B)}$ **B** $\frac{P(A \cap B)}{P(A)}$

C $\frac{P(A \cup B)}{P(A)}$ **D** $\frac{P(A \cap B)}{P(B)}$

12 Differentiate:

a $y = x^9 - 4x^2 + 7x + 3$ **b** $y = 2x(x^2 - 1)$ **c** $y = 3x^{-4}$

d $y = \frac{5}{2x^5}$ **e** $y = \sqrt{x^3}$ **f** $y = (2x + 3)^7$

g $y = \frac{1}{(x^2 - 7)^4}$ **h** $y = \sqrt[3]{5x + 1}$ **i** $y = \frac{5x^2 - 1}{2x + 3}$

13 Sketch the graph of:

a $y = \frac{4}{2x - 4}$ **b** $P(x) = x^3 + x^2 - 2x$ **c** $y = |x - 1|$

d $x^2 + y^2 = 25$ **e** $f(x) = -\sqrt{1 - x^2}$

14 In a class of 25 students, 11 play guitar, 9 play the piano, while 8 don't play either instrument. If one student is selected at random from the class, find the probability that this student will play:

a both guitar and piano

b neither guitar or piano

c only guitar.

15 The volume in litres of a rectangular container that is leaking over time t minutes is given by $V = -t^2 + 4t + 100$. Find:

a the initial volume

b the volume after 10 minutes

c the rate of change in volume after 10 minutes

d how long it will take, to 1 decimal place, until the container is empty.

16 **a** Find the equation of the tangent to the curve $y = x^3 - 3x$ at the point $P = (-2, -2)$.

b Find the equation of the normal to $y = x^3 - 3x$ at P.

c Find the point Q where this normal cuts the x-axis.

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17 Two dice are thrown. Find the probability of throwing:

a double 1 **b** any double **c** at least one 3

d a total of 6 **e** a total of at least 8.

18 The function $f(x) = ax^2 + bx + c$ has a tangent at $(1, -3)$ with a gradient of -1. It also passes through $(4, 3)$. Find the values of a, b and c.

19 Find the equation of the circle with centre $(-2, -3)$ and radius 5 units.

20 Find the centre and radius of the circle with equation:

a $x^2 + 6x + y^2 - 10y - 15 = 0$ **b** $x^2 + 10x + y^2 - 6y + 30 = 0$

21 $f(x) = 3x^2 - 4x + 9$.

a Find $f(x + h) - f(x)$.

b Show by differentiating from first principles that $f'(x) = 6x - 4$.

22 **a** Find the equation of the tangent to the curve $y = x^3 - 2$ at the point $P(1, -1)$.

b The curve $y = x^3 - 2$ meets the y-axis at Q. Find the equation of PQ.

c Find the equation of the normal to $y = x^3 - 2$ at the point $(-1, -3)$.

d Find the point R where this normal cuts the x-axis.

23 100 cards are numbered from 1 to 100. If one card is chosen at random, find the probability of selecting:

a an even number less than 30

b an odd number or a number divisible by 9.

24 A bag contains 5 white, 6 yellow and 3 blue balls. Two balls are chosen at random from the bag without replacement. Find the probability of choosing:

a 2 blue balls **b** a white ball and a yellow ball

25 If Scott buys 10 tickets, find the probability that he wins both first and second prizes in a raffle in which 100 tickets are sold.

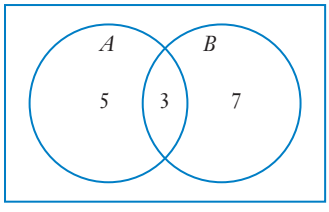
26 Two dice are rolled. Find the probability of rolling a total:

a of 8 **b** less than 7 **c** greater than 9

d of 4 or 5 **e** that is an odd number.

27 For the Venn diagram, find:

a $P(A|B)$ **b** $P(B|A)$



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28 A bag contains 5 red, 7 blue and 9 yellow balls. Cherylanne chooses 2 balls at random from the bag. Find the probability of that she chooses:

a blue given the first ball was yellow

b red given the first ball was blue.

29 If $f(x) = 2x^3 - 5x^2 + 4x - 1$, find $f(-2)$ and $f'(-2)$.

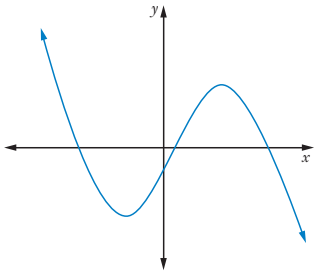
30 **a** Find the gradient of the secant to the curve $f(x) = 2x^3 - 7$ between the point $(2, 9)$ and the point where:

i $x = 2.01$ **ii** $x = 1.99$

b Hence estimate the gradient of the tangent to the curve at $(2, 9)$.

31 Sketch the gradient function for each curve.

a



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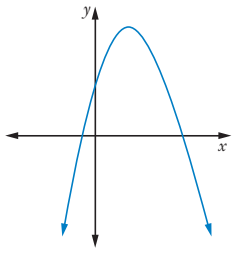

32 The area of a community garden in m^2 is given by $A = 7x - x^2$ where x is the length of the garden.

a Find the area when the length is:

i 3 m **ii** 4.5 m.

b Find the length when the area is 8 m^2, to 1 decimal place.

c Sketch the graph of the area function.

d Find the maximum possible area.

33 Solve graphically:

$$|x + 2| = 3.$$

34 If $f(x) = x^2 - 1$ and $g(x) = x^3 + 3$, find:

a the degree of $y = f(x)g(x)$

b the leading coefficient of $y = f(x)g(x)$

c the constant term of $y = f(x)g(x)$.

35 The displacement x cm of an object moving along a straight line over time t seconds is given by $x = 2t^3 - 13t^2 + 17t + 12$.

a Find the initial displacement, velocity and acceleration.

b Find the displacement, velocity and acceleration after 2 seconds.

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36 If $A = \{1, 3, 4, 5\}$ and $B = \{2, 3, 5, 6\}$:

a find $A \cup B$

b find $A \cap B$

c draw a Venn diagram showing this information.

37 Find the equation of the tangent to the curve $y = 3x^2 - 6x + 7$ at the point $(2, 7)$.

38 Find the derivative of:

a $y = x^{-3}$ **b** $y = \sqrt{x^3}$ **c** $y = \dfrac{1}{x^2}$ **d** $y = \dfrac{(7x+4)^2}{3x-1}$

e $y = (5x^2 + 1)(2x - 3)^4$ **f** $y = (3x + 1)^5$ **g** $y = \sqrt{2x-1}$

39 $f(x) = x^2 - 2$ and $g(x) = 2x - 1$.

a Find the equation of:

i $y = f(x) + g(x)$ **ii** $y = f(x)g(x)$

iii $y = g(x) - f(x)$ **iv** $y = \dfrac{g(x)}{f(x)}$

b Sketch the graph of:

i $y = -f(x)$ **ii** $y = g(-x)$

iii $y = -g(-x)$

40 **a** Find the centre and radius of the circle $x^2 + 2x + y^2 - 6y - 6 = 0$.

b Find its domain and range.

41 Find the equation of the normal to the curve $y = x^2 - 4x + 1$ at the point $(3, -2)$.

42 Differentiate:

a $y = 2x^4 - 5x^3 + 3x^2 - x - 4$ **b** $y = \dfrac{1}{2x^5}$ **c** $y = \sqrt{x}$

d $y = (2x - 3)^7$ **e** $y = 3x^4(2x - 5)^7$ **f** $y = \dfrac{5x+7}{3x-2}$

43 If $f(x) = x^2 + 1$ and $g(x) = x - 3$, find the degree of:

a $f(x) + g(x)$ **b** $f(x)g(x)$

44 A coin is tossed and a die thrown. Find the probability of getting:

a a head and a 6 **b** a tail and an odd number.

45 Find the domain and range of:

a $y = x^3 + 1$ **b** $y = 1 - x^2$

c $x^2 + 4x + y^2 - 2y - 20 = 0$ **d** $y = \dfrac{4}{x+2}$

46 If $f(x) = x^3$ and $g(x) = 2x + 5$, find:

a $f(g(x))$ **b** $g(f(x))$

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47 The table below shows the results of an experiment in tossing 2 coins.

Result	Frequency
HH	24
HT	15
TH	38
TT	23

a Add a column for relative frequencies as fractions.

b From the table, find the probability of tossing:

i 2 tails **ii** a head and a tail in any order

c What is the theoretical probability of tossing:

i 2 tails? **ii** a head and a tail in any order?

48 Find the equation of the tangent to the curve $y = x^3 - 7x + 3$ at the point where $x = 2$.

49 Find in exact form:

a the length of the arc

b the area of the sector

cut off by an angle of 40° at the centre of a circle with radius 4 cm.

50 If $f(x) = |x| - 2$ find:

a $f(-2)$ **b** $f(0)$ **c** $f(m + 1)$

51 The probability that Despina passes her first maths test is 64% and the probability that she will pass both the first and second tests is 48%. Find the probability that Despina passes the second test given that she passes the first test.

52 If $P(L) = 45\%$, $P(L \cap M) = 5.4\%$ and $P(M) = 12\%$, show that L and M are independent.

53 Given $P(X) = 0.26$, $P(Y) = 0.15$ and $P(X \cup Y) = 0.371$, show that X and Y are independent.

54 State whether events A and B are mutually exclusive if $P(A) = 0.18$, $P(A \cup B) = 0.5$ and $P(B) = 0.32$.

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