Practice set 4

For Questions **1** to **5**, select the correct answer **A**, **B**, **C** or **D**.

1 Find the amplitude and period of $y = 5 \sin 3x$.

A Amplitude 3, period 5 **B** Amplitude 5, period 3

C Amplitude 5, period $\frac{2\pi}{3}$ **D** Amplitude 3, period $\frac{2\pi}{5}$

2 The table is a discrete probability distribution.

x	1	2	3	4	5	6
$P(X = x)$	0.14	0.16	0.08	0.14	0.31	0.17

Find $P(X \leq 4)$.

A 0.38 **B** 0.52 **C** 0.14 **D** 0.62

3 Find the exact value of $\sin 135° + \cos 120°$.

A $\frac{\sqrt{2} - \sqrt{3}}{2}$ **B** $\frac{\sqrt{2} + 1}{2}$

C $\frac{\sqrt{2} + \sqrt{3}}{2}$ **D** $\frac{\sqrt{2} - 1}{2}$

4 Which statement is the same as $3^x = 7$? There is more than one answer.

A $x = \log \frac{7}{3}$ **B** $\log_3 x = 7$

C $\log_3 7 = x$ **D** $x = \frac{\log 7}{\log 3}$

5 The derivative of $x^2(2x + 9)^2$ is:

A $4x(2x + 9)$ **B** $2x(2x + 9)^2 + 2x^2(2x + 9)$

C $2x(2x + 9)$ **D** $2x(2x + 9)^2 + 4x^2(2x + 9)$

6 Differentiate:

a $y = e^x - x$ **b** $y = 3e^x + 1$ **c** $y = (e^x - 2)^4$

d $y = e^x(4x + 1)^3$ **e** $y = \frac{e^x}{5x - 2}$ **f** $y = 5e^{7x}$

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7 A function is given by

$$f(x) = \begin{cases} \dfrac{x+1}{8} & \text{for } x = 0, 1, 2 \\ \dfrac{x-2}{4} & \text{for } x = 3 \end{cases}$$

a Find:

i $f(0)$ **ii** $f(3)$

b Show that $f(x)$ is a probability function.

8 Find $\log_5 \dfrac{1}{25}$.

9 The table represents a probability distribution.

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$

Find:

a $P(X = 2)$ **b** $P(X < 4)$ **c** $P(X \geq 2)$

d $P(4 \leq X \leq 6)$ **e** $P(1 \leq X < 5)$

10 Simplify:

a $\tan(180° - \theta)$ **b** $\sin(-\theta)$ **c** $\cos(2\pi - \theta)$

11 For $0 \leq x \leq 2\pi$ sketch the graph of:

a $y = 2\sin 4x$ **b** $y = \tan \dfrac{x}{2}$ **c** $y = -\cos x$

12 For each random variable X, write the set of possible values.

a The number of rolls of a die until a 6 turns up

b The number of red cards selected when choosing 12 cards from a bag containing 15 red and 15 black cards

c The first rainy day in January.

13 Solve $\log_x \dfrac{1}{16} = 4$.

14 The population of a city over t years is given by the formula $P = 100\,000e^{0.71t}$. After how many years, to 1 decimal place, will the population become 1 million?

15 A bag contains 7 white and 6 blue cards. Create a probability distribution table for the number of blue cards selected when randomly selecting 3 cards:

a with replacement **b** without replacement.

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16 If $\tan x = -\frac{4}{3}$ and $\cos x > 0$, evaluate $\sin x$ and $\cos x$.

17 Solve for $0 \leq x \leq 2\pi$:

a $2\cos x + 1 = 0$ **b** $\tan^2 x = 1$

c $\cos x = 0$ **d** $\sin 2x = \frac{1}{2}$

18 This table represents a probability distribution.

x	1	2	3	4	5
$P(X = x)$	0.16	0.23	0.22	a	b

If $E(X) = 3.04$, evaluate a and b.

19 Find the expected value, variance and standard deviation for the probability distribution below.

x	0	1	2	3	4
$P(X = x)$	0.2	0.1	0.3	0.1	0.3

20 Find the exact value of:

a $\cos \frac{7\pi}{4}$ **b** $\sin \frac{4\pi}{3}$ **c** $\tan \frac{5\pi}{6}$

21 Draw a discrete probability distribution table for the number of tails when tossing 3 coins.

22 Sketch the graph of:

a $y = \log_3 x$ **b** $y = 3\log_2 x - 1$

23 **a** Write $\log_e x$ as an equation with x in terms of y.

b Hence find the value of x, to 3 significant figures, when $y = 1.23$.

24 Solve $7^{2x} = 3$.

25 This table shows a discrete probability distribution. Evaluate k.

x	0	1	2	3	4
$P(X = x)$	$2k$	$3k$	$4k - 2$	$5k - 1$	$6k$

26 State whether each probability distribution is uniform.

a Number of heads when tossing 2 coins

b Number of heads when tossing a coin

c Number of even numbers when rolling one die

d Number of 1s when rolling one die.

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27 State whether each function is a probability function.

a $f(x) = \dfrac{x+1}{10}$ for $x = 0, 1, 2, 3$

b $f(x) = \begin{cases} \dfrac{x}{11} & \text{for } x = 1, 2 \\ \dfrac{x-1}{22} & \text{for } x = 3, 4, 5 \end{cases}$

28 Solve for $0° \leq x \leq 360°$:

a $\tan x = -1$ **b** $2 \sin x = 1$

c $2 \cos^2 x = 1$ **d** $\tan 2x = \sqrt{3}$

29 Evaluate, to 2 decimal places where appropriate.

a $\log_2 16$ **b** $\log_3 3$ **c** $\log_4 2$

d $\log_{10} 109.7$ **e** $\ln 43.1$ **f** $\log_3 11$

30 Sketch the graph of:

a $y = e^{-x}$

b $y = 2e^{3x} + 1$

31 The probability of winning a game is 65% and the probability of losing the game is 12%.

a Draw a probability distribution table showing 0 for a loss, 1 for a draw and 2 for a win.

b Find the expected value and variance.

32 Find the equation of the tangent to the curve $y = 5e^x$ at the point $(2, 5e^2)$.

33 In a game, Faizal pays \$1 to toss 2 coins. He wins \$2 for 2 heads or 2 tails and loses \$1 for a head and a tail.

a Find the expected value for this game.

b How much would you expect Faizal to win or lose in the long term?

34 A spinner has the numbers 1 to 8 equally placed around it.

a Draw a probability distribution table for the spinner.

b Is it a uniform distribution?

c Find the probability of spinning a number:

i greater than 4 **ii** 3 or less **iii** at least 4

d Find the expected value of the spinner.

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35 **a** Show that the points (1, 27%), (2, 31%), (3, 28%) and (4, 14%) represent a discrete probability function.

b Find $E(X)$ and $Var(X)$.

36 For the following probability distribution, evaluate k.

x	1	2	3	4	5	6
$p(x)$	$\frac{5}{16}$	k	$\frac{1}{16}$	$\frac{3}{8}$	$\frac{1}{16}$	$\frac{1}{8}$

37 Simplify:

a $5 + 5\tan^2 x$ **b** $\dfrac{(1+\sin x)(1-\sin x)}{\sin x \cos x}$

38 Find the exact value of:

a $\tan 150°$ **b** $\cos(-45°)$ **c** $\sin 240°$

39 Find the value of x:

a $x^2 - 2x - 3 = 0$ **b** $1 < 2x - 3 \leq 7$ **c** $|3x + 1| = 4$

40 Find the centre and radius of the circle $x^2 - 4x + y^2 + 6y - 3 = 0$.

41 Amanda leaves home and cycles south for 3.6 km. She then turns and cycles for 5.4 km on a bearing of 243°.

a How far is Amanda from her house, to 1 decimal place?

b What is Amanda's bearing from her house, to the nearest degree?

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