4 Right-angled triangles

Syllabus topic — M3 Right-angled triangles

This topic is focused on solving problems involving right-angled triangles in a variety of contexts.

Outcomes

- Find unknown sides using Pythagoras' theorem.
- Solve problems using Pythagoras' theorem.
- Define the trigonometric ratios – sine, cosine and tangent.
- Use a calculator in trigonometry with angles to the nearest minute.
- Find unknown sides using trigonometry.
- Find unknown angles using trigonometry.
- Using trigonometry to solve practical problems.
- Solve problems involving compass and true bearings.
- Solve problems involving angles of elevation and depression.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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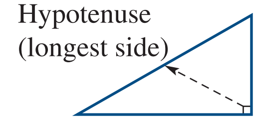
4A Pythagoras' theorem

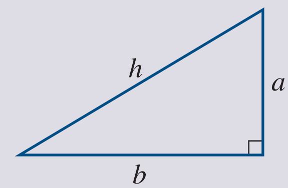
Pythagoras' theorem links the sides of a right-angled triangle. In a right-angled triangle the side opposite the right angle is called the hypotenuse. The hypotenuse is always the longest side.



PYTHAGORAS' THEOREM

Pythagoras' theorem states that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

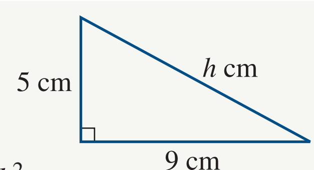
$(\text{Hypotenuse})^2 = (\text{side})^2 + (\text{other side})^2 \qquad h^2 = a^2 + b^2$



Pythagoras' theorem is used to find a missing side of a right-angled triangle if two of the sides are given. It can also be used to prove that a triangle is right angled.

Example 1: Finding the length of the hypotenuse 4A

Find the length of the hypotenuse, correct to two decimal places.



SOLUTION:

1 Write Pythagoras' theorem. $h^2 = a^2 + b^2$

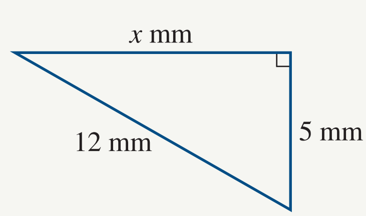
2 Substitute the length of the sides. $= 9^2 + 5^2$

3 Take the square root to find h. $h = \sqrt{9^2 + 5^2}$

4 Express the answer correct to two decimal places. $\approx 10.30\,\text{cm}$

Example 2: Finding the length of a shorter side 4A

What is the value of x, correct to one decimal place?



SOLUTION:

1 Write Pythagoras' theorem. $h^2 = a^2 + b^2$

2 Substitute the length of the sides. $12^2 = x^2 + 5^2$

3 Make x^2 the subject. $x^2 = 12^2 - 5^2$

4 Take the square root to find x. $x = \sqrt{12^2 - 5^2}$

5 Express the answer to correct one decimal place. $\approx 10.9\,\text{mm}$

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Exercise 4A

Example 1 **1** Find the length of the hypotenuse, correct to one decimal place.

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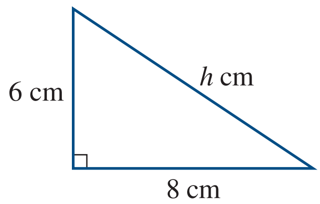

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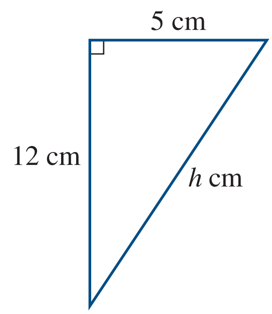

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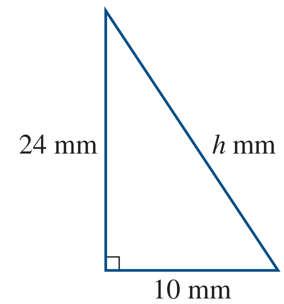

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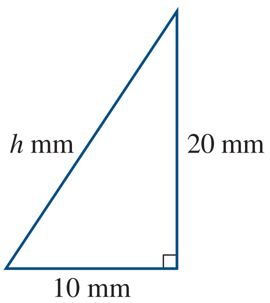

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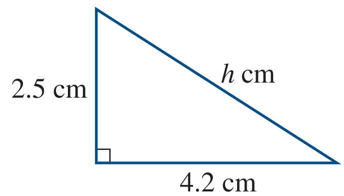

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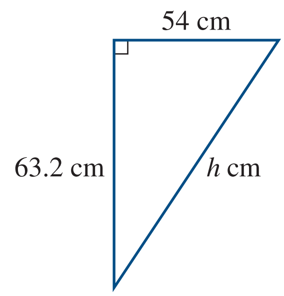

Example 2 **2** Find the value of x, correct to two decimal places.

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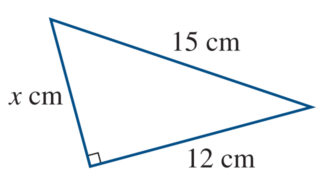

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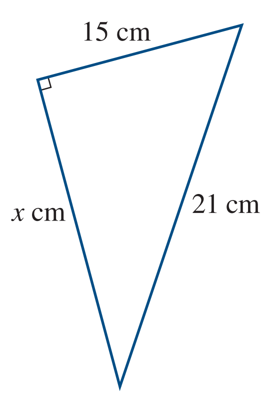

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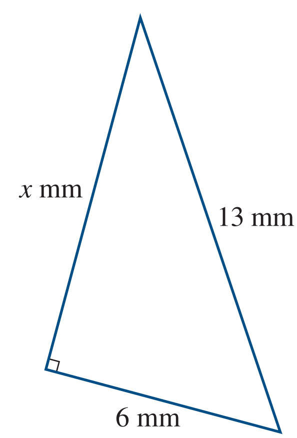

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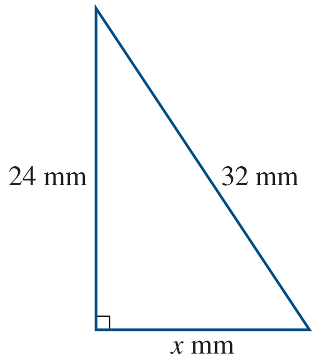

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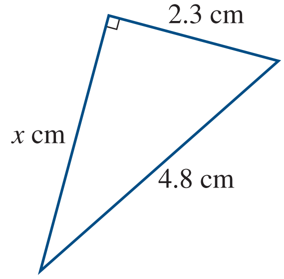

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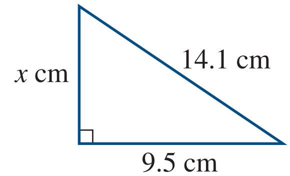

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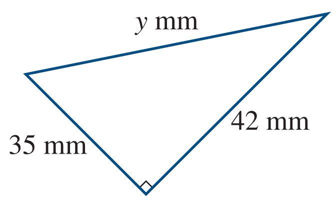
3 Calculate the length of the side marked with the pronumeral. (Answer to the nearest millimetre.)

a



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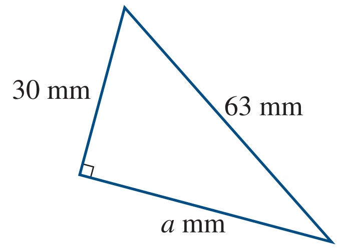

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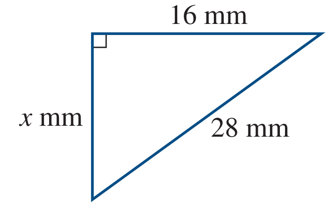

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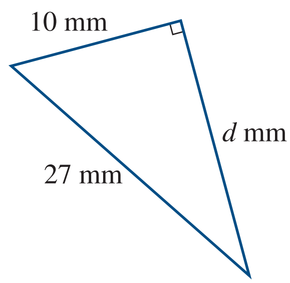

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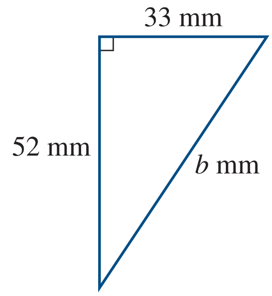

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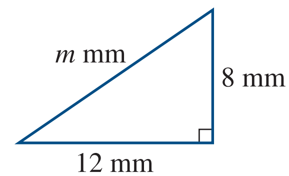

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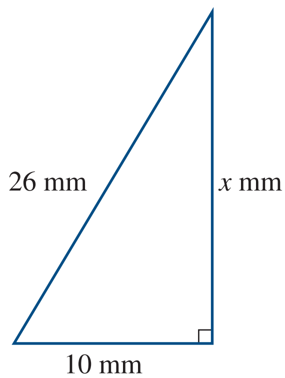

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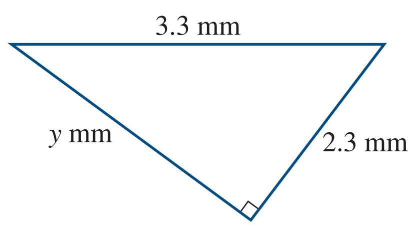

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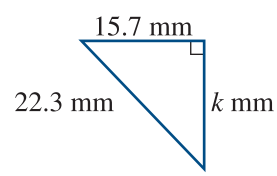
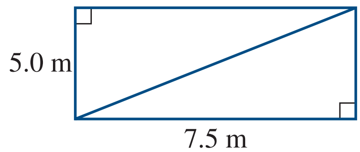
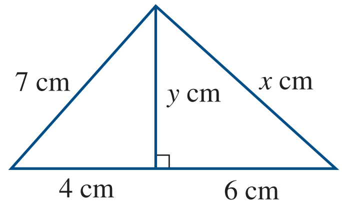


4 Find, correct to one decimal place, the length of the diagonal of a rectangle with dimensions 7.5 metres by 5.0 metres.



5 **a** Find the value of y, correct to two decimal places.

b Find the value of x using the value of y from part **a**, correct to two decimal places.



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4B Applying Pythagoras' theorem

Pythagoras' theorem is used to solve many practical problems. These problems are represented by a right-angle triangle and require the use of Pythagoras' theorem to determine the hypotenuse or the length of a shorter side.

SOLVING A PROBLEM USING PYTHAGORAS' THEOREM

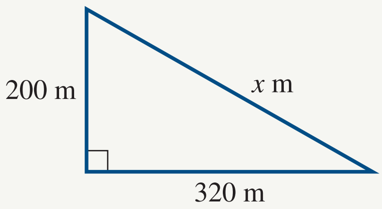
1. Read the question and underline the key terms.
2. Draw a diagram and label the information from the question.
3. Decide whether to determine the hypotenuse or the length of a shorter side.
4. Use Pythagoras' theorem to calculate a solution.
5. Check that the answer is reasonable and units are correct.
6. Explain the answer in words and ensure the question has been answered.

Example 3: Solving problems using Pythagoras' theorem 4B

A helicopter is at a height of 200 m above the ground and is a horizontal distance of 320 m from a landing pad. Find the direct distance of the helicopter from the landing pad, correct to two decimal places.

SOLUTION:

1. Draw a diagram and label the information from the question.
2. Label the hypotenuse with x. This represents the distance of the helicopter from the landing pad.



3. Write Pythagoras' theorem.
4. Substitute the length of the sides.
5. Take the square root to find x.
6. Express the answer correct to two decimal places.
7. Write your answer in words.

$$x^2 = a^2 + b^2$$
$$= 320^2 + 200^2$$
$$x = \sqrt{320^2 + 200^2}$$
$$\approx 377.36\,\text{m}$$

∴ The helicopter is 377.36 metres from the landing pad.

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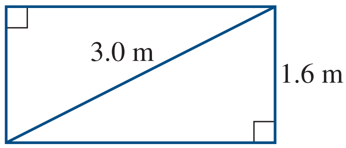
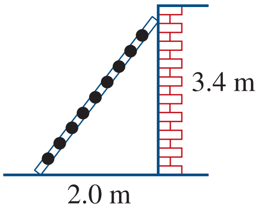
Exercise 4B

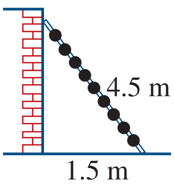
Example 3 **1** A farm gate that is 1.6 m high is supported by a diagonal bar of length 3.0 m. Find the width of the gate, correct to one decimal place.



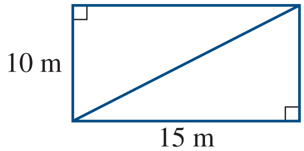
2 A ladder rests against a brick wall as shown in the diagram on the right. The base of the ladder is 2.0 m from the wall, and reaches 3.4 m up the wall. Find the length of the ladder, correct to one decimal place.



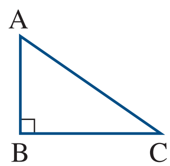
3 The base of a ladder leaning against a wall is 1.5 m from the base of the wall. If the ladder is 4.5 m long, find how high the top of the ladder is from the ground, correct to one decimal place.



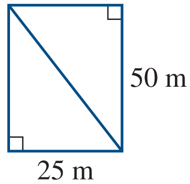
4 Find, correct to one decimal place, the length of the diagonal of a rectangle with dimensions 15 metres by 10 metres.



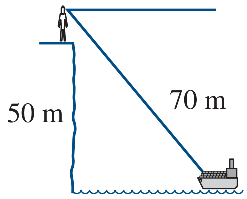
5 In a triangle ABC, there is a right angle at B. AB is 12 cm and BC is 16 cm. Find the length of AC.



6 A rectangular block of land measures 25 m by 50 m. John wants to put a fence along the diagonal. How long will the fence be? (Answer correct to three decimal places.)



7 Use the measurements on the diagram to determine the distance the boat is out to sea. (Answer correct to the nearest metre.)



8 The hypotenuse of a right-angled triangle is 40 cm long and one of the shorter sides measures 20 cm. What is the length of the remaining side in the triangle? (Answer correct to two decimal places.)

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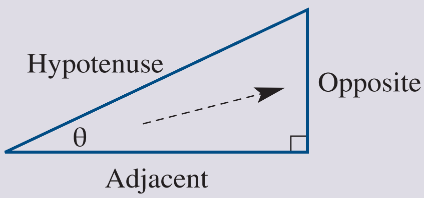
4C Trigonometric ratios

Trigonometric ratios are defined using the sides of a right-angled triangle. The hypotenuse is opposite the right angle, the opposite side is opposite the angle θ and the adjacent side is the remaining side.

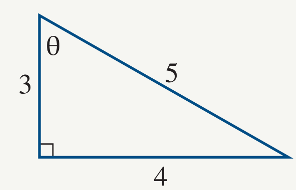
NAMING SIDES OF A RIGHT-ANGLED TRIANGLE



The opposite and adjacent sides are located in relation to the position of angle θ. If θ was in the other angle, the sides would swap their labels. The letter θ is the Greek letter theta. It is commonly used to label an angle.

Example 4: Naming the sides of a right-angled triangle 4C

What are the values of the hypotenuse, the opposite side and the adjacent side of the triangle shown?



SOLUTION:

1. Hypotenuse is opposite the right angle. — Hypotenuse is 5 ($h = 5$)
2. Opposite side is opposite the angle θ. — Opposite side is 4 ($o = 5$)
3. Adjacent side is beside the angle θ, but not the hypotenuse. — Adjacent side is 3 ($a = 5$)

The trigonometric ratios

The trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$ are defined using the sides of a right-angled triangle.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin\theta = \frac{o}{h}\ \text{(SOH)} \qquad \cos\theta = \frac{a}{h}\ \text{(CAH)} \qquad \tan\theta = \frac{o}{a}\ \text{(TOA)}$$

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TRIGONOMETRIC RATIOS

The mnemonic 'SOH CAH TOA' is pronounced as a single word.

SOH: **S**ine-**O**pposite-**H**ypotenuse

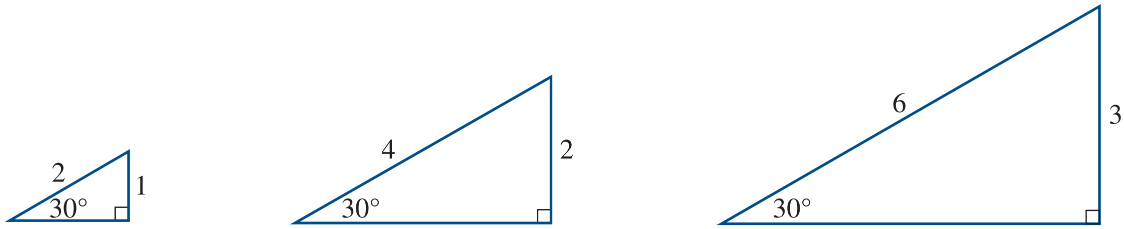
CAH: **C**osine-**A**djacent-**H**ypotenuse

TOA: **T**angent-**O**pposite-**A**djacent

The order of the letters matches the ratio of the sides.

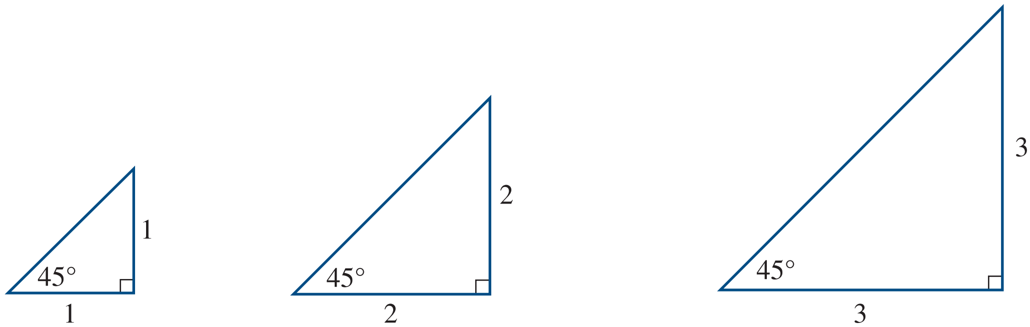
The meaning of the trigonometric ratios

Consider the three triangles drawn below.



The three triangles drawn above show the ratio of the opposite side to the hypotenuse as 0.5. $\left(\frac{1}{2}, \frac{2}{4} \text{ or } \frac{3}{6}\right)$. This is called the sine ratio. All right-angled triangles with an angle of 30° have a sine ratio of 0.5. If the angle is not 30° the ratio will be different, but any two right-angled triangles with the same angle will have the same value for their sine ratio.

Similarly, the three triangles drawn below show the ratio of the opposite side to the adjacent side as 1 $\left(\frac{1}{1}, \frac{2}{2} \text{ or } \frac{3}{3}\right)$. This is called the tangent ratio. All right-angled triangles with an angle of 45° have a tangent ratio of 1.



The ratio of the opposite side to the hypotenuse (sine ratio), the ratio of the adjacent side to the hypotenuse (cosine ratio) and the ratio of the opposite side to the adjacent side (tangent ratio) will always be constant irrespective of the size of the right-angled triangle.

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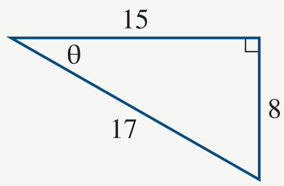
Example 5: Finding the trigonometric ratio 4C

Find the sine, cosine and tangent ratios for angle θ in the triangle shown.



SOLUTION:

1 Name the sides of the right-angled triangle. $a\ (15),\ o\ (8),\ h\ (17)$

2 Write the sine ratio (SOH).

3 Substitute the values for the opposite side and the hypotenuse.

$$\sin\theta = \frac{o}{h} = \frac{8}{17}$$

4 Write the cosine ratio (CAH).

5 Substitute the values for the adjacent side and the hypotenuse.

$$\cos\theta = \frac{a}{h} = \frac{15}{17}$$

6 Write the tangent ratio (TOA).

7 Substitute the values for the adjacent side and the opposite side.

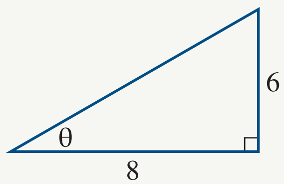
$$\tan\theta = \frac{o}{a} = \frac{8}{15}$$

Example 6: Finding a trigonometric ratio 4C

Find $\sin\theta$ in simplest form given $\tan\theta = \frac{6}{8}$.

SOLUTION:

1 Draw a triangle and label the opposite and adjacent sides.



2 Find the hypotenuse using Pythagoras' theorem.

3 Substitute the length of the sides into Pythagoras' theorem.

$$h^2 = 6^2 + 8^2$$

4 Take the square root to find the hypotenuse (h).

$$h = \sqrt{6^2 + 8^2}$$

5 Evaluate.

$$= 10$$

6 Write the sine ratio (SOH).

$$\sin\theta = \frac{o}{h}$$

7 Substitute the values for the opposite side and the hypotenuse.

$$= \frac{6}{10}$$

8 Simplify the ratio.

$$= \frac{3}{5}$$

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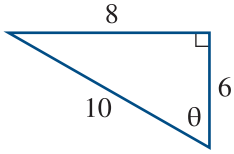
Exercise 4C

Example 4 **1** State the values of the hypotenuse, opposite side and adjacent side in each triangle.

a


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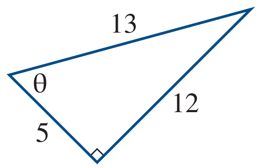

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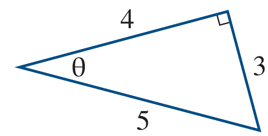

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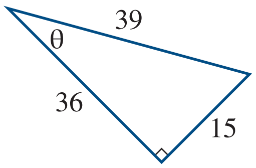

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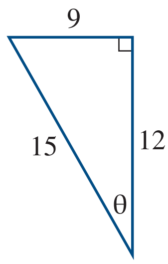

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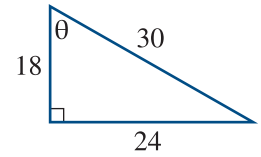
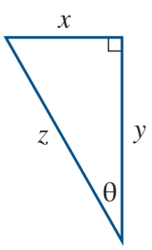


2 State the values of the hypotenuse, opposite side and adjacent side in each triangle.

a


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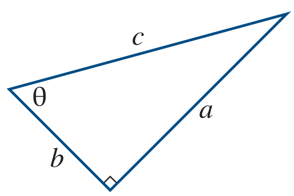

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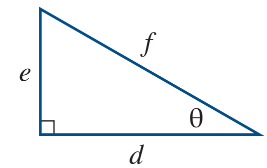
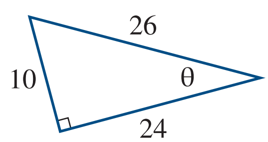


Example 5 **3** Write the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$ for each triangle in question **1**.

4 Write the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$ for each triangle in question **2**.

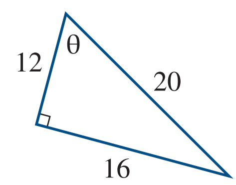
5 Name the trigonometric ratio represented by the following fractions.

a


i $\frac{10}{26}$

ii $\frac{24}{26}$

iii $\frac{10}{24}$

b


i $\frac{12}{20}$

ii $\frac{16}{12}$

iii $\frac{16}{20}$

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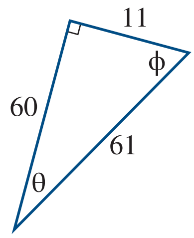
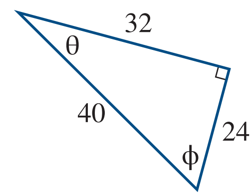
6 Find the sine, cosine and tangent ratios in simplest form for each angle.

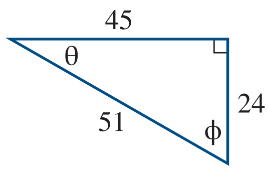
a **i** angle θ
ii angle ϕ



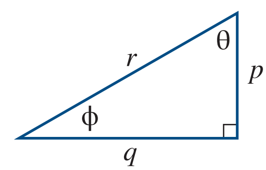
b **i** angle θ
ii angle ϕ



c **i** angle θ
ii angle ϕ

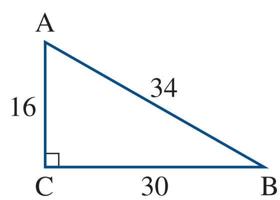


d **i** angle θ
ii angle ϕ

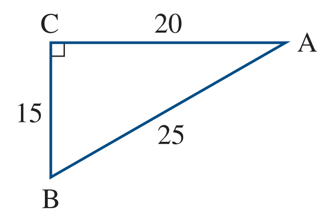


7 Find the sine and cosine ratios in simplest form for angle A and B for each triangle.

a



b



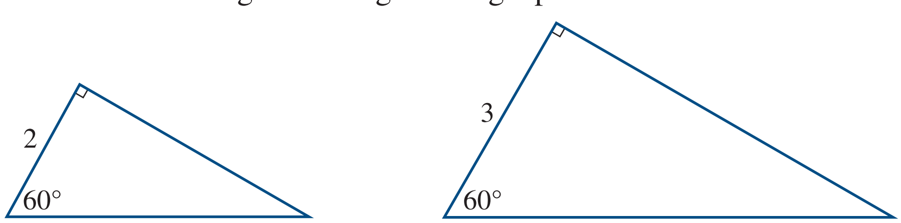
Example 6 **8** Draw a right-angled triangle for each of the following trigonometric ratios and
i find the length of the third side
ii find the other two trigonometric ratios in simplest form.

a $\tan\theta = \dfrac{3}{4}$

b $\sin\theta = \dfrac{8}{10}$

c $\cos\theta = \dfrac{7}{25}$

9 Draw the following two triangles using a protractor and a ruler.



a Measure the length of the hypotenuse, adjacent and opposite sides in each triangle.
b What is the value of the cosine ratio for 60° in both triangles?
c What is the value of the sine ratio for 60° in both triangles?

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4D Using the calculator in trigonometry

In trigonometry, an angle is usually measured in degrees, minutes and seconds. Make sure the calculator is set up to accept angles in degrees. It is essential in this course that the degree mode is selected.

DEGREES	MINUTES
1 degree = 60 minutes	1 minute = 60 seconds
$1^\circ = 60'$	$1' = 60''$

Finding a trigonometric ratio

A calculator is used to find a trigonometric ratio of a given angle. It requires the $\boxed{\text{sin}}$, $\boxed{\text{cos}}$ and $\boxed{\text{tan}}$ keys. The trigonometric ratio key is pressed followed by the angle. The degrees, minutes and seconds $\boxed{{}^\circ\,'\,''}$ or $\boxed{\text{DMS}}$ is then selected to enter minutes and seconds. Some calculators may require you to choose degrees, minutes and seconds from an options menu.

Example 7: Finding a trigonometric ratio — 4D

Find the value of the following, correct to two decimal places.

a $\sin 60^\circ$ **b** $2\cos 40.5^\circ$ **c** $\dfrac{3}{\tan 75^\circ}$ **d** $\sin 34^\circ 20'$

SOLUTION:

1 Press $\boxed{\text{sin}}$ 60 $\boxed{=}$ or $\boxed{\text{exe}}$

a $\sin 60^\circ = 0.866\,025\,403\,8$
≈ 0.87

2 Press 2 $\boxed{\text{cos}}$ 40.5 $\boxed{=}$ or $\boxed{\text{exe}}$

b $2\cos 40.5^\circ = 1.520\,811\,31$
≈ 1.52

3 Press 3 ÷ $\boxed{\text{tan}}$ 75 $\boxed{=}$ or $\boxed{\text{exe}}$

c $\dfrac{3}{\tan 75^\circ} = 0.803\,847\,577\,3$
≈ 0.80

4 Press $\boxed{\text{sin}}$ 34 $\boxed{{}^\circ\,'\,''}$ 20 $\boxed{{}^\circ\,'\,''}$ $\boxed{=}$ or $\boxed{\text{exe}}$

d $\sin 34^\circ 20' = 0.564\,006\,558\,1$
≈ 0.56

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Finding an angle from a trigonometric ratio

A calculator is used to find a given angle from a trigonometric ratio. Check that the degree mode is selected. To find an angle, use the $\boxed{\sin^{-1}}$, $\boxed{\cos^{-1}}$ and $\boxed{\tan^{-1}}$ keys. To select these keys, press the SHIFT or a 2nd function key. The degrees, minutes and seconds $\boxed{\circ\,\prime\,\prime\prime}$ or $\boxed{\text{DMS}}$ is then selected to find the angle in minutes and seconds.

Example 8: Finding an angle from a trigonometric ratio — 4D

a Given $\sin\theta = 0.6123$, find the value of θ to the nearest degree.
b Given $\tan\theta = 1.45$, find the value of θ to the nearest minute.

SOLUTION:

1 Press SHIFT $\boxed{\sin^{-1}}$ 0.6123 $\boxed{=}$ or $\boxed{\text{exe}}$.

a $\sin\theta = 0.6123$
$\theta = 37.755\,994\,38$
$\approx 38^\circ$

2 Press SHIFT $\boxed{\tan^{-1}}$ 1.45 $\boxed{=}$ or $\boxed{\text{exe}}$.
Convert the answer to minutes by using the $\boxed{\circ\,\prime\,\prime\prime}$ or $\boxed{\text{DMS}}$.

b $\tan\theta = 1.45$
$\theta = 55^\circ 24' 27.76''$
$\approx 55^\circ 24'$

Example 9: Finding an angle from a trigonometric ratio — 4D

a Given $\sin\theta = \frac{4}{5}$, find the value of θ to the nearest degree.

b Given $\cos\theta = \frac{1}{\sqrt{2}}$, find the value of θ to the nearest degree.

SOLUTION:

1 Press SHIFT $\boxed{\sin^{-1}}$ $\frac{4}{5}$ $\boxed{=}$ or $\boxed{\text{exe}}$.

a $\sin\theta = \frac{4}{5}$
$\theta = 53.130102\ldots \approx 53^\circ$

2 Press SHIFT $\boxed{\cos^{-1}}$ $\frac{1}{\sqrt{2}}$ $\boxed{=}$ or $\boxed{\text{exe}}$.

b $\cos\theta = \frac{1}{\sqrt{2}}$
$\theta = 45^\circ$

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Exercise 4D

1 What is the value of the following angles in minutes?

a $1°$ **b** $3°$ **c** $5°$ **d** $7°$
e $10°$ **f** $15°$ **g** $20°$ **h** $60°$
i $0.5°$ **j** $\frac{1}{3}°$ **k** $0.2°$ **l** $0.25°$

2 What is the value of the following angles in degrees?

a 120 minutes **b** 480 minutes
c 60 minutes **d** 600 minutes
e 360 minutes **f** 240 minutes
g 900 minutes **h** 720 minutes
i 30 minutes **j** 15 minutes
k 45 minutes **l** 20 minutes

Example 7a

3 Find the value of the following trigonometric ratios, correct to two decimal places.

a $\sin 20°$ **b** $\cos 43°$
c $\tan 65°$ **d** $\cos 72°$
e $\tan 13°$ **f** $\sin 82°$
g $\cos 15°$ **h** $\tan 48°$

Example 7b

4 Find the value of the following trigonometric ratios, correct to two decimal places.

a $\cos 63°30'$ **b** $\sin 40°10'$ **c** $\cos 52°45'$ **d** $\tan 35°23'$
e $\sin 22°56'$ **f** $\tan 53°42'$ **g** $\tan 68°2'$ **h** $\cos 65°57'$

Example 7c

5 Find the value of the following trigonometric ratios, correct to one decimal place.

a $4\cos 30°$ **b** $3\tan 53°$ **c** $5\sin 74°$ **d** $6\sin 82°$
e $6\tan 77°$ **f** $2\cos 43°$ **g** $8\sin 12°$ **h** $9\tan 54°$

6 Find the value of the following trigonometric ratios, correct to one decimal place.

a $4\sin 65°20'$ **b** $5\tan 23°55'$ **c** $12\cos 10°41'$ **d** $8\sin 21°9'$
e $11\sin 21°30'$ **f** $7\cos 32°40'$ **g** $4\sin 25°12'$ **h** $8\tan 39°24'$

Example 7d

7 Find the value of the following trigonometric ratios, correct to two decimal places.

a $\frac{5}{\tan 40°}$ **b** $\frac{1}{\sin 63°}$ **c** $\frac{12}{\cos 25°}$ **d** $\frac{3}{\sin 42°}$
e $\frac{4}{\cos 38°9'}$ **f** $\frac{5}{\tan 72°36'}$ **g** $\frac{6}{\sin 55°48'}$ **h** $\frac{7}{\cos 71°16'}$

Example 8a

8 Given the following trigonometric ratios, find the value of θ to the nearest degree.

a $\sin\theta = 0.5673$ **b** $\cos\theta = 0.1623$ **c** $\tan\theta = 0.2782$
d $\cos\theta = 0.7843$ **e** $\tan\theta = 0.5047$ **f** $\sin\theta = 0.1298$

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Example 8b **9** Given the following trigonometric ratios, find the value of θ to the nearest minute.

a $\tan\theta = 0.3891$ **b** $\sin\theta = 0.6456$
c $\cos\theta = 0.1432$ **d** $\sin\theta = 0.8651$
e $\cos\theta = 0.3810$ **f** $\tan\theta = 0.8922$

Example 9a **10** Given the following trigonometric ratios, find the value of θ to the nearest degree.

a $\tan\theta = \dfrac{3}{4}$ **b** $\sin\theta = \dfrac{1}{2}$
c $\cos\theta = \dfrac{5}{8}$ **d** $\cos\theta = \dfrac{1}{4}$
e $\sin\theta = \dfrac{3}{5}$ **f** $\tan\theta = 1\dfrac{1}{3}$

Example 9b **11** Given the following trigonometric ratios, find the value of θ to the nearest degree.

a $\sin\theta = \dfrac{\sqrt{3}}{2}$ **b** $\tan\theta = \dfrac{1}{\sqrt{5}}$
c $\cos\theta = \dfrac{\sqrt{5}}{6}$ **d** $\tan\theta = \dfrac{4}{\sqrt{6}}$
e $\cos\theta = \dfrac{\sqrt{3}}{2}$ **f** $\sin\theta = \dfrac{1}{\sqrt{2}}$

12 Given the following trigonometric ratios, find the value of θ to the nearest minute.

a $\cos\theta = \dfrac{2}{\sqrt{7}}$ **b** $\sin\theta = \dfrac{\sqrt{3}}{4}$
c $\tan\theta = \dfrac{\sqrt{5}}{12}$ **d** $\sin\theta = \dfrac{\sqrt{7}}{7}$
e $\tan\theta = \dfrac{\sqrt{2}}{7}$ **f** $\cos\theta = \dfrac{3}{\sqrt{11}}$

13 Given that $\sin\theta = 0.4$ and angle θ is less than 90°, find the value of:

a θ to the nearest degree
b $\cos\theta$ correct to one decimal place
c $\tan\theta$ correct to two decimal places.

14 Given that $\cos\theta = 0.8$ and angle θ is less than 90°, find the value of:

a θ to the nearest degree
b $\sin\theta$ correct to one decimal place
c $\tan\theta$ correct to two decimal places.

15 Given that $\tan\theta = 2.1$ and angle θ is less than 90°, find the value of:

a θ to the nearest minute
b $\sin\theta$ correct to three decimal places
c $\cos\theta$ correct to four decimal places.

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4E Finding an unknown side

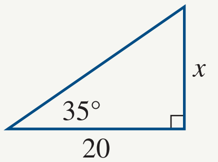
Trigonometric ratios are used to find an unknown side in a right-angled triangle, given at least one angle and one side. The method involves labelling the sides of the triangle and using the mnemonic SOH CAH TOA. The resulting equation is rearranged to make x the subject and the calculator used to find the unknown side.

FINDING AN UNKNOWN SIDE IN A RIGHT-ANGLED TRIANGLE

1. Name the sides of the triangle – h for hypotenuse, o for opposite and a for adjacent.
2. Use the given side and unknown side x to determine the trigonometric ratio. The mnemonic SOH CAH TOA helps with this step.
3. Rearrange the equation to make the unknown side x the subject.
4. Use the calculator to find x. Remember to check the calculator is set up for degrees.
5. Write the answer to the specified level of accuracy.

Example 10: Finding an unknown side 4E

Find the length of the unknown side x in the triangle shown. Answer correct to three decimal places.

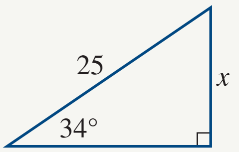


SOLUTION:

1. Name the sides of the right-angled triangle. $a\ (20), o\ (x), h$
2. Determine the ratio (TOA). $\tan\theta = \dfrac{o}{a}$
3. Substitute the known values. $\tan 35° = \dfrac{x}{20}$
4. Multiply both sides of the equation by 20. $x = 20 \times \tan 35°$
5. Press 20 $\boxed{\tan}$ 35 $\boxed{=}$ $= 14.004150...$
6. Write the answer correct to three decimal places. ≈ 14.004

Example 11: Finding an unknown side 4E

Find the length of the unknown side x in the triangle shown. Answer correct to two decimal places.



SOLUTION:

1. Name the sides of the right-angled triangle. $a, o\ (x), h\ (25)$
2. Determine the ratio (SOH). $\sin\theta = \dfrac{o}{h}$
3. Substitute the known values. $\sin 34° = \dfrac{x}{25}$
4. Multiply both sides of the equation by 25. $x = 25 \times \sin 34°$
5. Press 25 $\boxed{\sin}$ 34 $\boxed{=}$ $= 13.979822...$
6. Write the answer correct to two decimal places. ≈ 13.98

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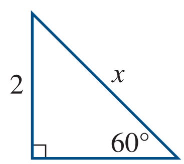
Finding an unknown side in the denominator

It is possible that the unknown side (x) is the denominator of the trigonometric ratio. For example, in the triangle below, the unknown x is the hypotenuse of the triangle. This results in the trigonometric ratio $\frac{2}{x}$.

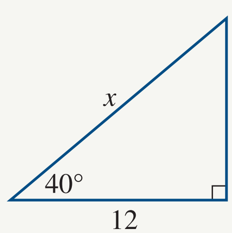


$$\sin\theta = \frac{o}{h}$$
$$\sin 60^\circ = \frac{2}{x}$$

To solve these types of equations, multiply both sides by x. Then divide both sides by the trigonometric expression ($\sin 60^\circ$) to make x the subject.

Example 12: Finding an unknown side in the denominator 4E

Find the length of the unknown side x in the triangle shown. Answer correct to two decimal places.



SOLUTION:

1. Name the sides of the right-angled triangle. — a (12), o, h (x)
2. Determine the ratio (CAH). — $\cos\theta = \frac{a}{h}$
3. Substitute the known values. — $\cos 40^\circ = \frac{12}{x}$
4. Multiply both sides of the equation by x. — $x \times \cos 40^\circ = 12$
5. Divide both sides by $\cos 40^\circ$. — $x = \frac{12}{\cos 40^\circ}$
6. Press 12 ÷ $\boxed{\cos}$ 40 $\boxed{=}$ or $\boxed{\text{exe}}$ — $x = 15.664\,887\,47$
7. Write the answer correct to two decimal places. — ≈ 15.66

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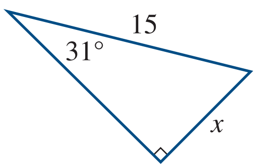
Exercise 4E

Example 10 **1** Find the length of the unknown side x in each triangle, correct to two decimal places.

a


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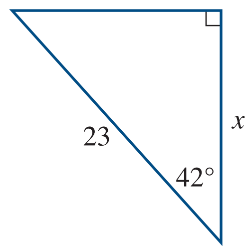

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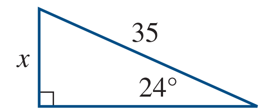

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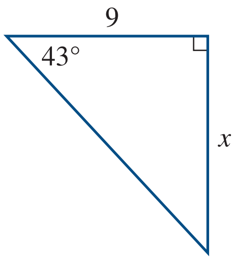

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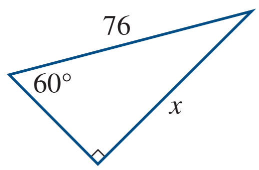

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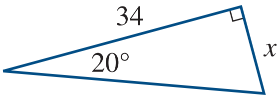

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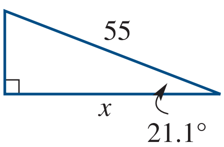

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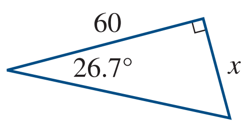
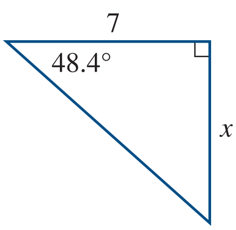
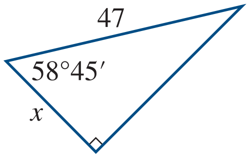


i


Example 11 **2** Find the length of the unknown side x in each triangle, correct to two decimal places.

a


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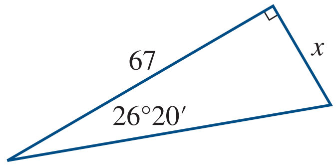

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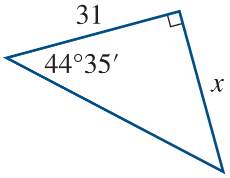

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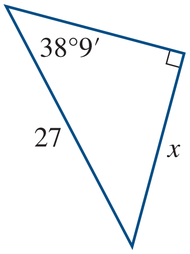

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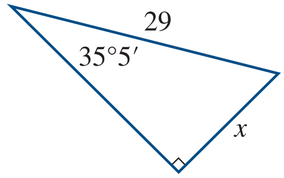

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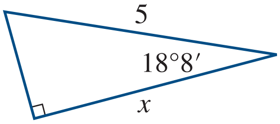

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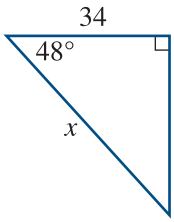
Example 12 **3** Find the length of the unknown side x in each triangle, correct to two decimal places.

a


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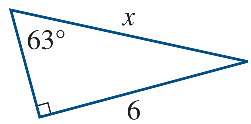

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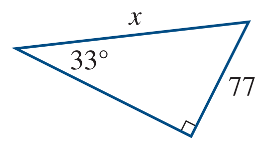

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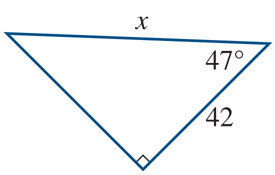

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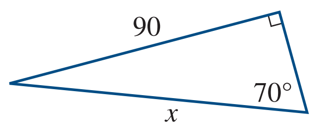

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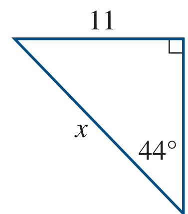
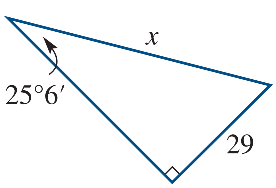


4 Find the length of the unknown side x in each triangle, correct to one decimal place.

a


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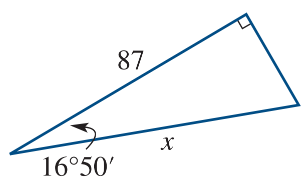

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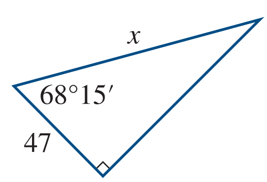

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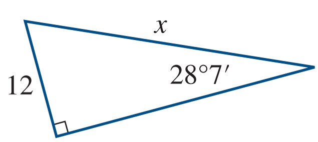

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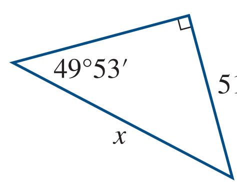

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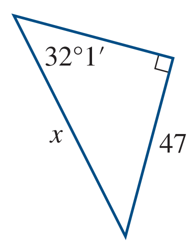
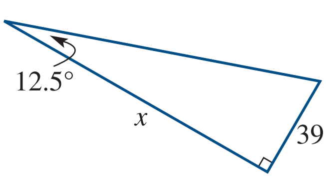


5 Find the length of the unknown side x in each triangle, correct to three decimal places.

a


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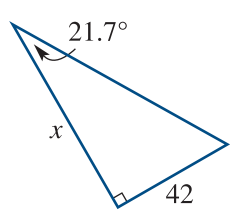

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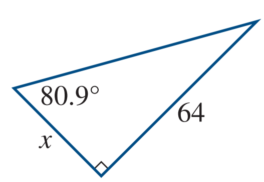

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4F Finding an unknown angle

Trigonometric ratios are used to find an unknown angle in a right-angled triangle, given at least two sides. The method involves labelling the sides of the triangle and using the mnemonic SOH CAH TOA. The resulting equation is rearranged to make θ the subject and the calculator is used to find the unknown angle.

FINDING AN UNKNOWN ANGLE IN A RIGHT-ANGLED TRIANGLE

1. Name the sides of the triangle – h for hypotenuse, o for opposite and a for adjacent.
2. Use the given sides and unknown angle θ to determine the trigonometric ratio. The mnemonic SOH CAH TOA helps with this step.
3. Rearrange the equation to make the unknown angle θ the subject.
4. Use the calculator to find θ. Remember to check the calculator is set up for degrees.
5. Write the answer to the specified level of accuracy.

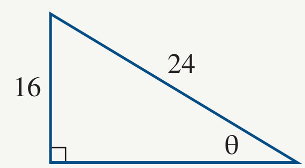
Example 13: Finding an unknown angle 4F

Find the angle θ in the triangles shown:

a to the nearest degree.

b to the nearest minute.

a



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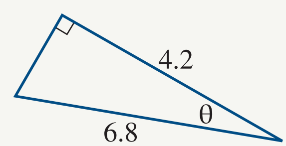

SOLUTION:

1 Name the sides of the right-angled triangle.	**a** h (24), o (16), a
2 Determine the ratio (SOH).	$\sin\theta = \frac{o}{h}$
3 Substitute the known values.	$\sin\theta = \frac{16}{24}$
4 Make θ the subject of the equation.	$\theta = \sin^{-1}\left(\frac{16}{24}\right)$
5 Press SHIFT $\boxed{\sin^{-1}}$ (16 ÷ 24) $\boxed{=}$ or $\boxed{\text{exe}}$	$= 41.8103149$
or Press SHIFT $\boxed{\sin^{-1}}$ 16 $\boxed{a^{b}/_{c}}$ 24 $\boxed{=}$ or $\boxed{\text{exe}}$	
6 Write the answer correct to the nearest degree.	$\theta = 42°$
7 Name the sides of the right-angled triangle.	**b** h (6.8), o, a (4.2)
8 Determine the ratio (CAH).	$\cos\theta = \frac{a}{h}$
9 Substitute the known values.	$\cos\theta = \frac{4.2}{6.8}$
10 Make θ the subject of the equation.	$\theta = \cos^{-1}\left(\frac{4.2}{6.8}\right)$
11 Press SHIFT $\boxed{\cos^{-1}}$ (4.2 ÷ 6.2) $\boxed{=}$	$= 51.855486\ldots$
or Press SHIFT $\boxed{\cos^{-1}}$ 4.2 $\boxed{a^{b}/_{c}}$ 6.8 $\boxed{=}$	
12 Write the answer correct to the nearest minute.	$\approx 51°51'$

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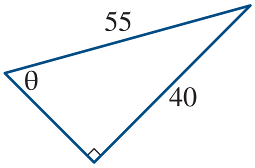
Exercise 4F

Example 13 **1** Find the unknown angle θ in each triangle. Answer correct to the nearest degree.

a


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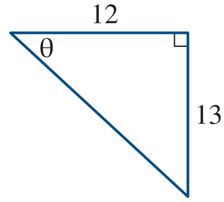

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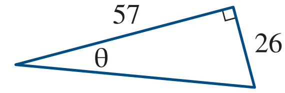

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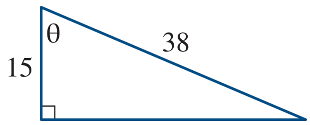

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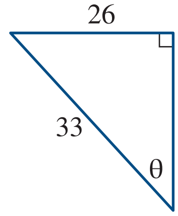

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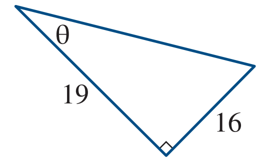

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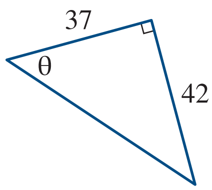

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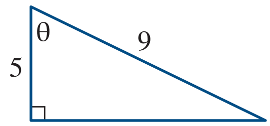

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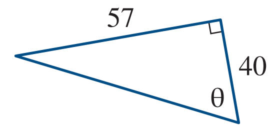
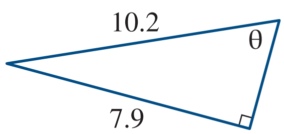


2 Find the unknown angle θ in each triangle. Answer correct to the nearest degree.

a


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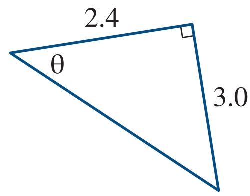

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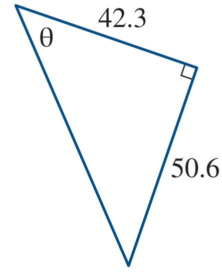

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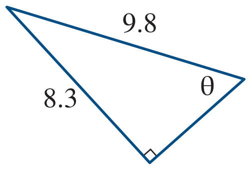

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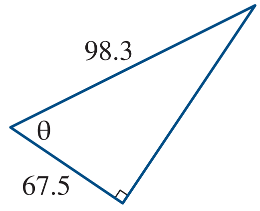

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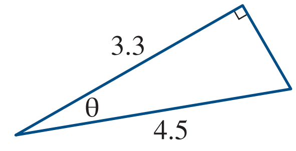

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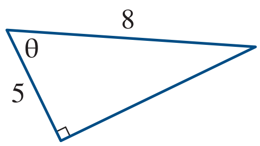
3 Find the unknown angle θ in each triangle. Answer correct to the nearest minute.

a



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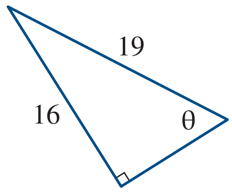

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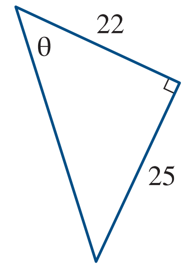

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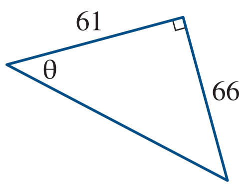

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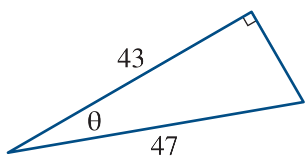

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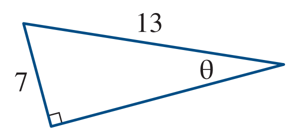
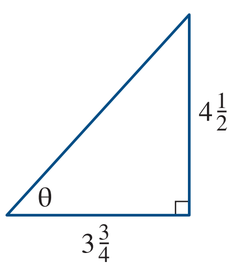


4 Find the unknown angle θ in each triangle. Answer correct to the nearest degree.

a



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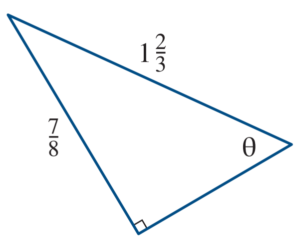

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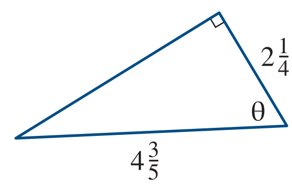
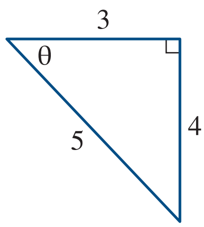


5 Find the unknown angle θ in each triangle. Answer correct to the nearest degree.

a



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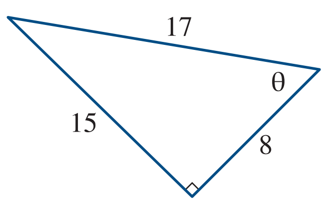

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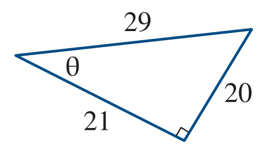
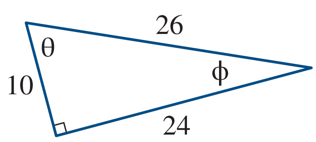


6 Find the angle θ and ϕ in each triangle. Answer correct to the nearest minute.

a



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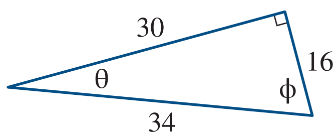

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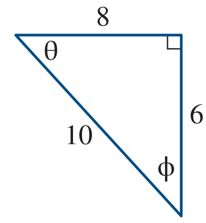

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4G Solving practical problems

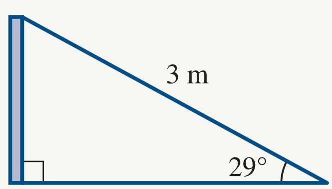
Trigonometry is used to solve many practical problems. How high is that tree? What is the height of the mountain? Calculate the width of the river. When solving a trigonometric problem, make sure you read the question carefully and draw a diagram. Label all the information given in the question on this diagram.

SOLVING A TRIGONOMETRIC WORDED PROBLEM

1 Read the question and underline the key terms.
2 Draw a diagram and label the information from the question.
3 Use trigonometry to calculate a solution.
4 Check that the answer is reasonable and units are correct.
5 Write the answer in words and ensure the question has been answered.

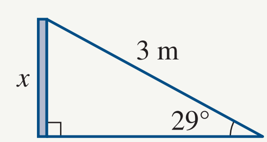
Example 14: Application requiring the length of a side — 4G

A vertical tent pole is supported by a rope tied to the top of the pole and to a peg on the ground. The rope is 3 m in length and makes an angle of 29° to the horizontal. What is the height of the tent pole? Answer correct to two decimal places.



SOLUTION:

1 Draw a diagram and label the required height as x.



2 Name the sides of the right-angled triangle. — a, o (x), h (3 m)

3 Determine the ratio (SOH). — $\sin\theta = \dfrac{o}{h}$

4 Substitute the known values. — $\sin 29° = \dfrac{x}{3}$

5 Multiply both sides of the equation by 3. — $x = 3 \times \sin 29°$

6 Press 3 $\boxed{\sin}$ 29 $\boxed{=}$ — $= 1.454428\ldots$

7 Write the answer correct to two decimal places. — ≈ 1.45

8 Write the answer in words. — $\therefore$ Height of the tent pole is 1.45 m.

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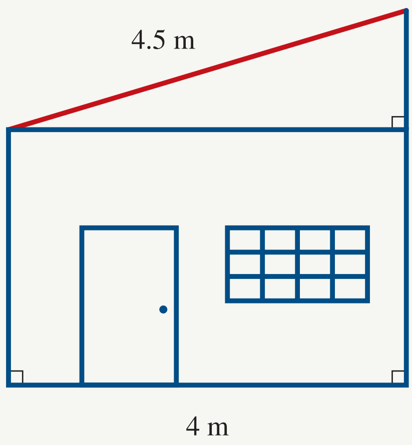
Applications requiring an angle

Trigonometry has many applications, such as in building and construction. Any vertical parts of a structure make a right angle with horizontal parts. Sloping lines in the structure complete a right-angled triangle, and trigonometry can be used to calculate its other angles and side lengths.

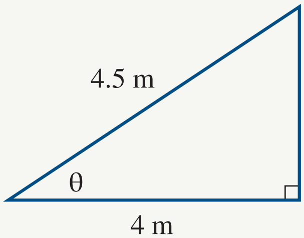
Example 15: Application requiring an angle 4G

The sloping roof of a shed uses sheets of Colorbond steel 4.5 m long on a shed 4 m wide. There is no overlap of the roof past the sides of the walls. Find the angle the roof makes with the horizontal. Answer correct to the nearest degree.



SOLUTION:

1 Draw a diagram and label the required angle as θ.



2 Name the sides of the right-angled triangle. a (4 m), o, h (4.5 m)

3 Determine the ratio (CAH). $\cos\theta = \dfrac{a}{h}$

4 Substitute the known values. $\cos\theta = \dfrac{4}{4.5}$

5 Make θ the subject of the equation. $\theta = \cos^{-1}\left(\dfrac{4}{4.5}\right)$

6 Press SHIFT $\boxed{\cos^{-1}}$ (4 ÷ 4.5) $\boxed{=}$ or $\boxed{\text{exe}}$ $= 27.26604445$

7 Write the answer correct to the nearest degree. $\theta \approx 27^\circ$

8 Write the answer in words. The roof makes an angle of 27°.

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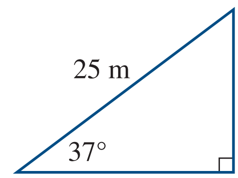
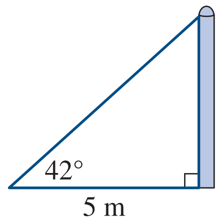
Exercise 4G

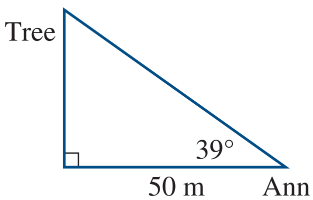
Example 14 **1** A balloon is tied to a string 25 m long. The other end of the string is secured by a peg to the surface of a level sports field. The wind blows so that the string forms a straight line making an angle of 37° with the ground. Find the height of the balloon above the ground. Answer correct to one decimal place.



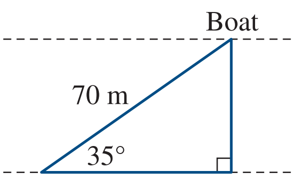
2 A pole is supported by a wire that runs from the top of the pole to a point on the level ground 5 m from the base of the pole. The wire makes an angle of 42° with the ground. Find the height of the pole, correct to two decimal places.



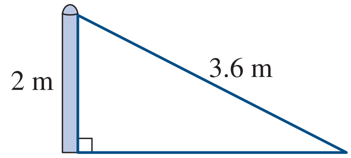
3 Ann noticed a tree was directly opposite her on the far bank of the river. After she walked 50 m along the side of the river, she found her line of sight to the tree made an angle of 39° with the river bank. Find the width of the river, to the nearest metre.



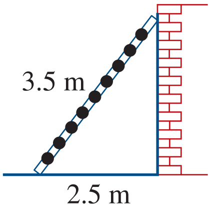
4 A ship at anchor requires 70 m of anchor chain. If the chain is inclined at 35° to the horizontal, find the depth of the water, correct to one decimal place.



Example 15 **5** A vertical tent pole is supported by a rope of length 3.6 m tied to the top of the pole and to a peg on the ground. The pole is 2 m in height. Find the angle the rope makes to the horizontal. Answer correct to the nearest degree.



6 A 3.5 m ladder has its foot 2.5 m out from the base of a wall. What angle does the ladder make with the ground? Answer correct to the nearest degree.



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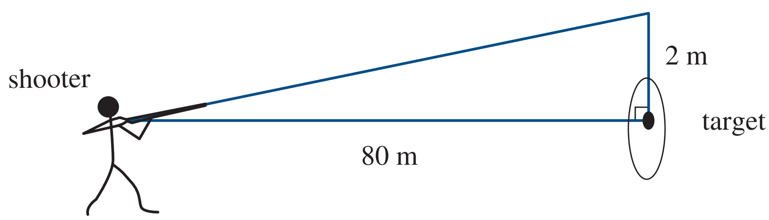
7 A plane maintains a flight path of 19° with the horizontal after it takes off. It travels for 4 km along the flight path. Find, correct to one decimal place:

a the horizontal distance of the plane from its take-off point

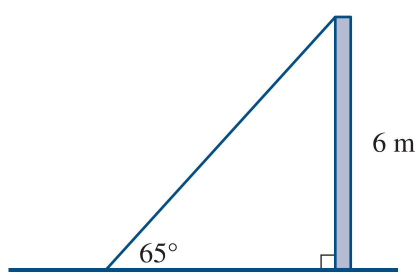
b the height of the plane above ground level.

8 A wheelchair ramp is being provided to allow access to the first floor shops. The first floor is 3 m above the ground floor. The ramp requires an angle of 20° with the horizontal. How long will the ramp be, measured along its slope? Answer correct to two decimal places.

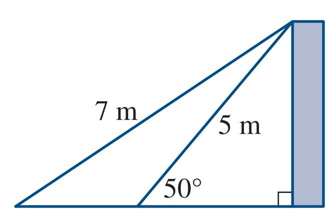
9 A shooter 80 m from a target and level with it, aims 2 m above the bullseye and hits it. What is the angle, to the nearest minute, that his rifle is inclined to the line of sight from his eye to the target?



10 A rope needs to be fixed with one end attached to the top of a 6 m vertical pole and the other end pegged at an angle of 65° with the level ground. Find the required length of rope. Answer correct to one decimal place.



11 Two ladders are the same distance up the wall. The shorter ladder is 5 m long and makes an angle of 50° with the ground. The longer ladder is 7 m long. Find:



a the distance the ladders are up the wall, correct to two decimal places

b the angle the longer ladder makes with the ground, correct to the nearest degree.

12 A pole is supported by a wire that runs from the top of the pole to a point on the level ground 7.2 m from the base of the pole. The height of the pole is 5.6 m. Find the angle, to the nearest degree, that the wire makes with the ground.

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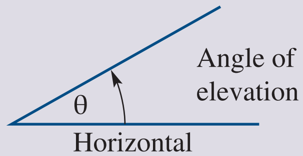
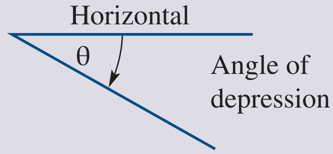
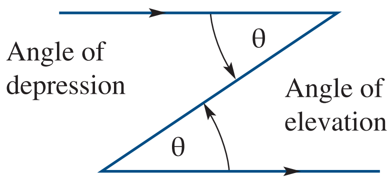
4H Angles of elevation and depression

The angle of elevation is the angle measured upwards from the horizontal. The angle of depression is the angle measured downwards from the horizontal.

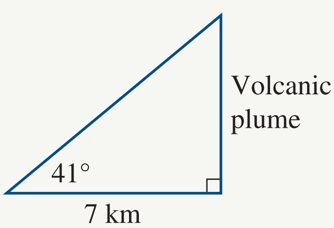
ANGLE OF ELEVATION	ANGLE OF DEPRESSION
	

The angle of elevation is equal to the angle of depression as they form alternate angles between two parallel lines. This information is useful to solve some problems.



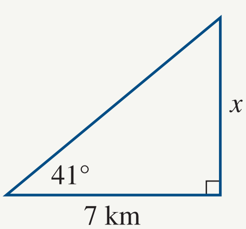
Example 16: Angle of elevation 4H

A park ranger measured the top of a plume of volcanic ash to be at an angle of elevation of 41°. From her map she noted that the volcano was 7 km away. Calculate the height of the plume of volcanic ash. Answer correct to two decimal places.



SOLUTION:

1 Draw a diagram and label the required height as x.



2 Determine the ratio (TOA).

$$\tan\theta = \frac{o}{a}$$

3 Substitute the known values.

$$\tan 41° = \frac{x}{7}$$

4 Multiply both sides of the equation by 7.

$$7 \times \tan 41° = x$$
$$x = 7 \times \tan 41°$$

5 Write the answer correct to two decimal places.

$$x = 6.085\,007\,165$$
$$\approx 6.09$$

6 Write the answer in words.

The height of the volcanic plume was 6.09 km.

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Example 17: Finding a distance using angle of depression 4H

The top of a cliff is 85 m above sea level. Minh saw a tall ship. He estimated the angle of depression to be 17°.

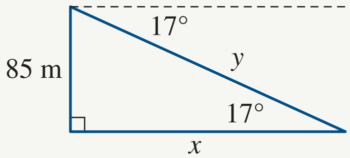
a How far was the ship from the base of the cliff? Answer to the nearest metre.

b How far is the ship in a straight line from the top of the cliff? Answer to the nearest metre.

SOLUTION:

1 Draw a diagram and label the distance to the base of the cliff as x and the distance to the top of the cliff as y.

a



2 Determine the ratio (TOA).

$$\tan\theta = \frac{o}{a}$$

3 Substitute the known values.

$$\tan 17° = \frac{85}{x}$$

4 Multiply both sides of the equation by x.

$$x \times \tan 17° = 85$$

5 Divide both sides by tan 17°.

$$x = \frac{85}{\tan 17°}$$

$$= 278.022...$$

6 Write the answer correct to nearest metre.

$$\approx 278\,\text{m}$$

7 Write the answer in words.

∴ The ship is 278 metres from the base of the cliff.

8 Determine the ratio (SOH).

b $\sin\theta = \frac{o}{h}$

9 Substitute the known values.

$$\sin 17° = \frac{85}{y}$$

10 Multiply both sides of the equation by y.

$$y \times \sin 17° = 85$$

11 Divide both sides by sin 17°.

$$y = \frac{85}{\sin 17°}$$

12 Write the answer correct to nearest metre.

$$= 290.7258...$$

$$\approx 291\,\text{m}$$

13 Write the answer in words.

∴ The ship is 291 metres from the top of the cliff.

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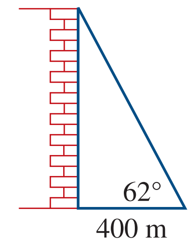
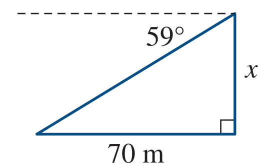
Exercise 4H

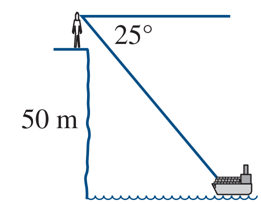
Example 16 **1** Luke walked 400 m away from the base of a tall building, on level ground. He measured the angle of elevation to the top of the building to be 62°. Find the height of the building. Answer correct to the nearest metre.



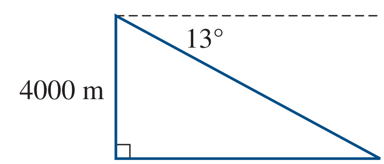
2 The angle of depression from the top of a TV tower to a satellite dish near its base is 59°. The dish is 70 m from the centre of the tower's base on flat land. Find the height of the tower. Answer correct to one decimal place.



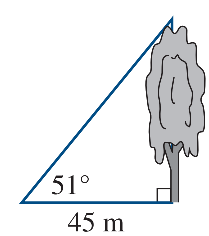
Example 17 **3** When Sarah looked from the top of a cliff 50 m high, she noticed a boat at an angle of depression of 25°. How far was the boat from the base of the cliff? Answer correct to two decimal places.



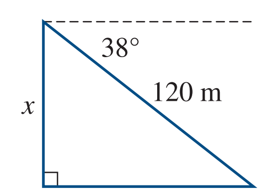
4 The pilot of an aeroplane saw an airport at sea level at an angle of depression of 13°. His altimeter showed that the aeroplane was at a height of 4000 m. Find the horizontal distance of the aeroplane from the airport. Answer correct to the nearest metre.



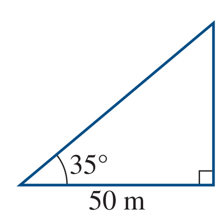
5 The angle of elevation to the top of a tree is 51° at a distance of 45 m from the point on level ground directly below the top of the tree. What is the height of the tree? Answer correct to one decimal place.



6 A iron ore seam of length 120 m slopes down at an angle of depression from the horizontal of 38°. The mine engineer wishes to sink a vertical shaft, x, as shown. What is the depth of the required vertical shaft? Answer correct to the nearest metre.



7 Jack measures the angle of elevation to the top of a tree from a point on level ground as 35°. What is the height of the tree if Jack is 50 m from the base of the tree? Answer to the nearest metre.



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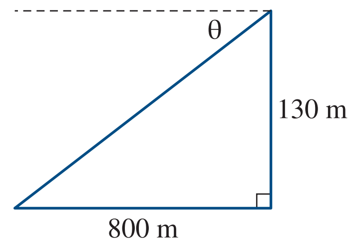
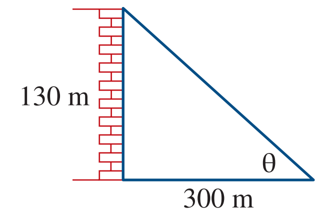
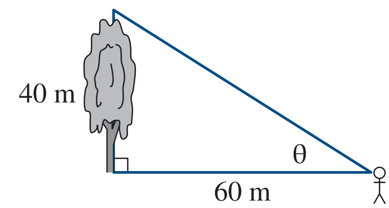
8 A tourist viewing Sydney Harbour from a building 130 m above sea level observes a ferry that is 800 m from the base of the building. Find the angle of depression. Answer correct to the nearest degree.



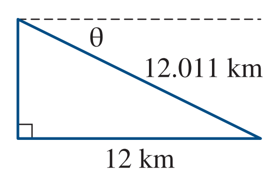
9 What would be the angle of elevation to the top of a radio transmitting tower 130 m tall and 300 m from the observer? Answer correct to the nearest degree.



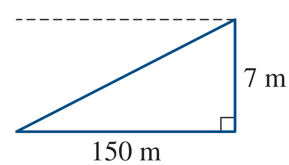
10 Lachlan observes the top of a tree at a distance of 60 m from the base of the tree. The tree is 40 m high. What is the angle of elevation to the top of the tree? Answer correct to the nearest degree.



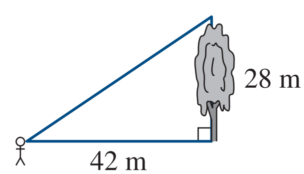
11 A town is 12 km from the base of a mountain. The town is also a distance of 12.011 km in a straight line to the mountain. What is the angle of depression from the top of a mountain to the town? Answer to the nearest degree, correct to one decimal place.



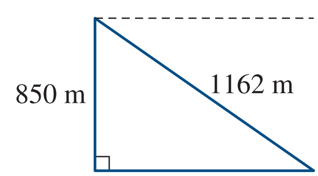
12 Find, to the nearest degree, the angle of elevation of a railway line that rises 7 m for every 150 m along the track.



13 The distance from the base of a tree is 42 m. The tree is 28 m in height. What is the angle of elevation measured from ground level to the top of a tree? Answer correct to the nearest degree.



14 A helicopter is flying 850 m above sea level. It is also 1162 m in a straight line to a ship. What is the angle of depression from the helicopter to the ship? Answer correct to the nearest degree.



15 The angle of elevation to the top of a tree from a point *A* on the ground is 25°. The point *A* is 22 m from the base of the tree. Find the height of the tree. Answer correct to nearest metre.

16 A plane is 460 m directly above one end of a 1200 m runway. Find the angle of depression to the far end of the runway. Answer correct to the nearest minute.

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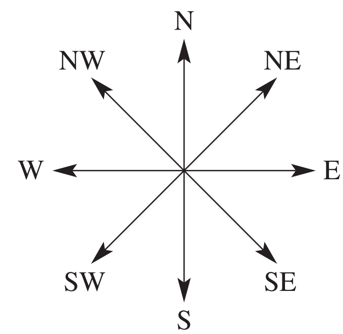
4I Compass and true bearings

A bearing is the direction one object is from another object or an observer or a fixed point. There are two types of bearings: compass bearings and true bearings.

Compass bearings

Compass bearings use the four directions of the compass: north, east, south and west (N, E, S and W). The NS line is vertical and the EW line is horizontal. In-between these directions are another four directions: north-east, south-east, south-west and north-west (NE, SE, SW and NW). Each of these directions makes an angle of 45° with the NS and EW lines.



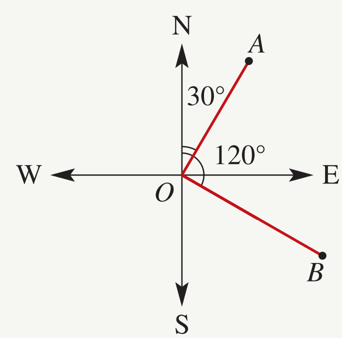
A direction is given using a compass bearing by stating the angle either side of north or south. For example, a compass bearing of S50°W is found by measuring an angle of 50° from the south direction towards the west side.

Example 18: Understanding a compass bearing — 4I

Find the compass bearing of:

a *A* from *O*

b *B* from *O*.



SOLUTION:

1 Determine the quadrant of the compass bearing.	**a** The line *OA* is in the north/east quadrant.
2 Find the angle the direction makes with the vertical (north/south) line.	30°
3 Write the compass bearing with N or S first, then the angle with the vertical line and finally either E or W.	Compass bearing of *A* from *O* is N30°E.
4 Determine the quadrant of the compass bearing.	**b** The line *OB* is in the south/east quadrant.
5 Find the angle the direction makes with the vertical (north/south) line.	$180° - 120° = 60°$
6 Write the compass bearing with N or S first, then the angle with vertical line and finally either E or W.	Compass bearing of *B* from *O* is S60°E.

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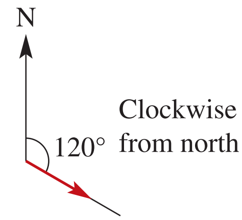
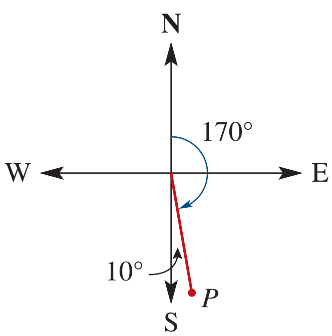
True bearings

A true bearing is the angle measured clockwise from north around to the required direction, and it is written with the letter T after the degree or minutes or seconds symbol. True bearings are sometimes called three-figure bearings because they are written using three numbers or figures. For example, 120°T is the direction measured 120° clockwise from north. It is the same bearing as S60°E.

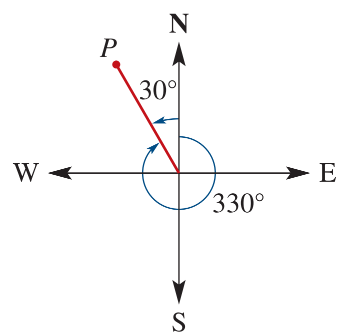


The smallest true bearing is 000°T and the largest true bearing is 360°T. The eight directions of the compass have the following true bearings: north is 000°T, east is 090°T, south is 180°T, west is 270°T, north-east is 045°T, south-east is 135°T, south-west is 225°T and north-west is 315°T.

The bearings in the following diagrams are given using both methods.



Compass bearing S 10°E True bearing 170°T



Compass bearing N 30°W True bearing 330°T

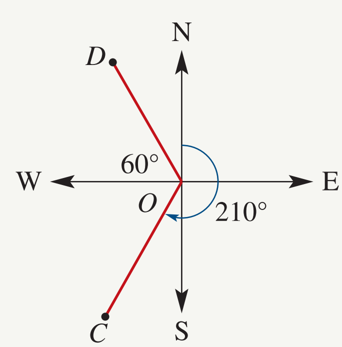
COMPASS BEARING	TRUE BEARING
A direction given by stating the angle either side of north or south, such as S60°E.	A direction given by measuring the angle clockwise from north to the required direction, such as 120°T.

Example 19: Understanding a true bearing — 4I

Find the true bearing of:

a C from O

b D from O.



SOLUTION:

1 Find the angle the bearing makes in the clockwise direction with the north direction.	**a** 210°
2 Write the true bearing using this angle. Add the letter 'T'.	C from O is 210°T.
3 Write the true bearing of west.	**b** 270°T
4 Add angle between west and D to true bearing for west.	270° + 60° = 330°
5 Write the true bearing using this sum. Add the letter 'T'.	D from O is 330°T.

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Exercise 4I

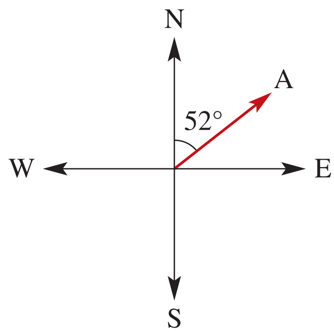
1 State the compass bearing and the true bearing of each of the following directions.

a NE **b** NW **c** SE **d** SW

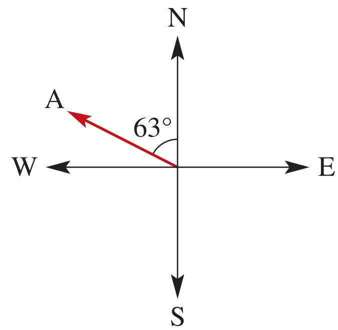
Example 18, 19

2 State the compass bearing and the true bearing of each of the following directions.

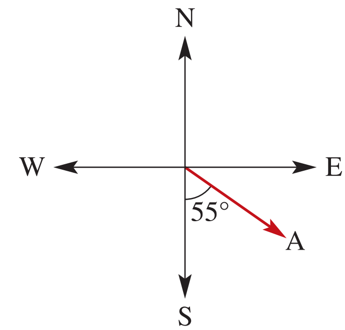
a



b



c



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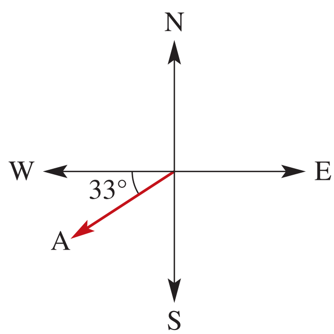

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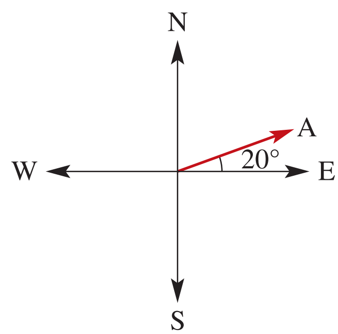

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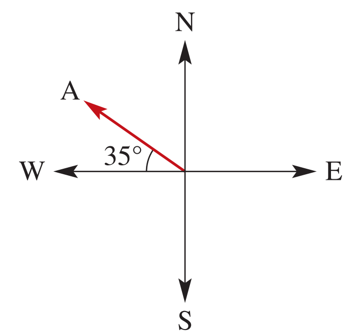

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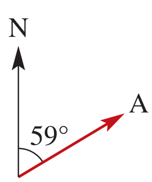

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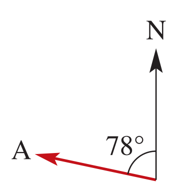

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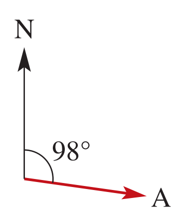

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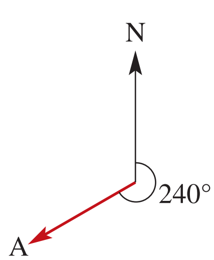

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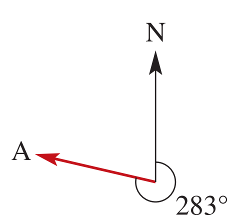

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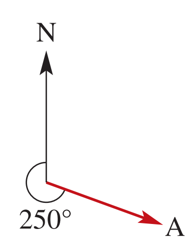
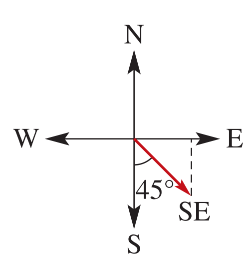


3 Sketch each of these bearings on a separate diagram.

a N10°E **b** S25°W **c** N60°W **d** S42°E

e 300° **f** 105° **g** 219° **h** 050°

4 Aaron runs a distance of 7.2 km in the SE direction. How far east has Aaron run? Answer correct to one decimal place.



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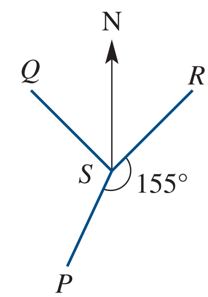
5 A plane is travelling on a true bearing of 030° from A to B.

- **a** What is the compass bearing of A to B?
- **b** What is the true bearing of B to A?
- **c** What is the compass bearing of B to A?

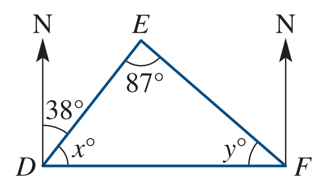
6 The diagram shows the position of P, Q and R relative to S. In the diagram, R is NE of S, Q is NW of S and $\angle PSR$ is 155°.

- **a** What is the true bearing of R from S?
- **b** What is the true bearing of Q from S?
- **c** What is the true bearing of P from S?



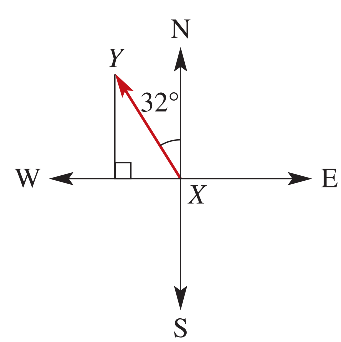
7 The bearing of E from D is $N38°E$, F is east of D and $\angle DEF$ is 87°

- **a** Find the values of x and y.
- **b** What is the compass bearing of E from F?
- **c** What is the true bearing of E from F?



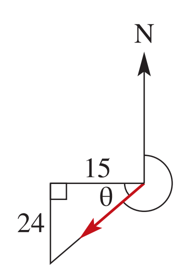
8 Riley travels from X to Y for 125 km on a bearing of $N32°W$.

- **a** How far did Riley travel due north, to the nearest kilometre?
- **b** How far did Riley travel due west, to the nearest kilometre?



9 Mia cycled for 15 km west and then 24 km south.

- **a** What is the value of θ to the nearest degree?
- **b** What is Mia's true bearing from her starting point?
- **c** What is Mia's compass bearing from her starting point?



10 A boat sails 137 km from Port Stephens on a bearing of 065°T.

- **a** How far east has the boat sailed? Answer correct to one decimal place.
- **b** How far north has the boat sailed? Answer correct to one decimal place.

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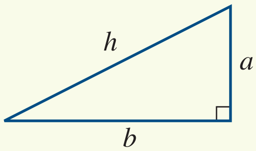
Summary

Key ideas and chapter summary

PPT STUDY GUIDE

Pythagoras' theorem

Pythagoras' theorem states that the square of the hypotenuse is equal to the sum of the squares of the other two sides ($h^2 = a^2 + b^2$).



Applying Pythagoras' theorem

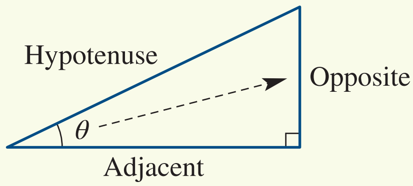
1. Read the question and underline the key terms.
2. Draw a diagram and label the information from the question.
3. Decide whether to determine the hypotenuse or the length of a shorter side.
4. Use Pythagoras' theorem to calculate a solution.
5. Check that the answer is reasonable and units are correct.

Trigonometric ratios

$\sin\theta = \dfrac{o}{h}$ (SOH)

$\cos\theta = \dfrac{a}{h}$ (CAH)

$\tan\theta = \dfrac{o}{a}$ (TOA)



Using the calculator in trigonometry

1 degree = 60 minutes $\quad 1° = 60'$

1 minute = 60 seconds $\quad 1' = 60''$

Finding an unknown side

1. Name the sides of the triangle.
2. Use the given side and unknown side x to determine the trigonometric ratio. The mnemonic SOH CAH TOA helps.
3. Rearrange the equation to make the unknown side x the subject then use the calculator to find x.

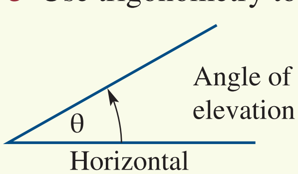
Finding an unknown angle

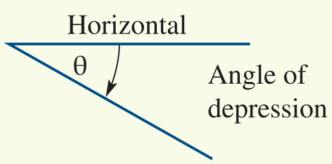
1. Name the sides of the triangle.
2. Use the given sides and unknown angle θ to determine the trigonometric ratio. The mnemonic SOH CAH TOA helps.
3. Rearrange the equation to make the unknown angle θ the subject then use the calculator to find θ.

Solving practical problems

1. Read the question and underline the key terms.
2. Draw a diagram and label the information from the question.
3. Use trigonometry to calculate a solution.

Angles of elevation and depression





Bearings

Compass bearing A direction given by stating the angle either side of north or south such as S60°E.

True bearing A direction given by measuring the angle clockwise from north to the required direction such as 120°.

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Review

Multiple-choice

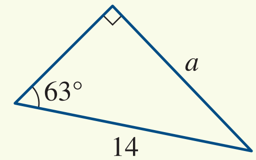
1 What is the length of the hypotenuse if the two other sides are 12 cm and 16 cm?

A 400 cm **B** 20 cm **C** 28 cm **D** 7.46 cm

2 What is the length of a?

A $14 \cos 63°$ **B** $14 \sin 63°$

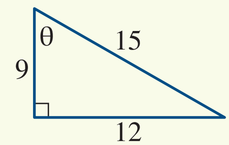
C $\dfrac{14}{\cos 63°}$ **D** $\dfrac{14}{\sin 63°}$



3 What is the value of $\cos \theta$?

A $\dfrac{9}{15}$ **B** $\dfrac{12}{9}$

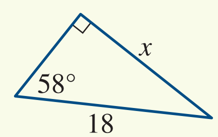
C $\dfrac{12}{15}$ **D** $\dfrac{9}{12}$



4 What is the length of x?

A $18 \cos 58°$ **B** $18 \sin 58°$

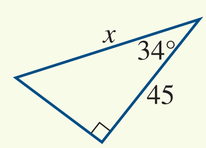
C $\dfrac{18}{\cos 58°}$ **D** $\dfrac{18}{\sin 58°}$



5 What is the length of x?

A $45 \cos 34°$ **B** $45 \sin 34°$

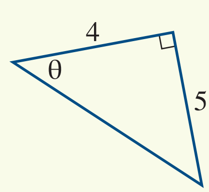
C $\dfrac{45}{\cos 34°}$ **D** $\dfrac{45}{\sin 34°}$



6 How would angle θ be calculated?

A $\tan^{-1}\left(\dfrac{5}{4}\right)$ **B** $\tan^{-1}\left(\dfrac{4}{5}\right)$

C $\tan\left(\dfrac{5}{4}\right)$ **D** $\tan\left(\dfrac{4}{5}\right)$



7 What is the size of angle θ in question **6**? (Answer correct to one decimal place.)

A 38.6° **B** 39.0° **C** 51.0° **D** 51.3°

8 What is the angle of elevation to the top of a tower 80 m tall and 100 m from the observer? Answer in degrees correct to one decimal place.

A 51.3° **B** 51.4° **C** 38.6° **D** 38.7°

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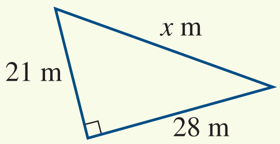
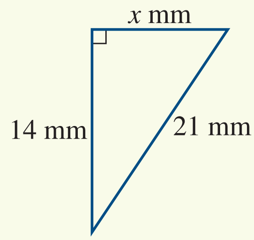
Short-answer

1 Find the value of x, correct to two decimal places.

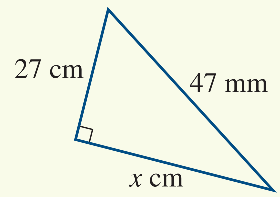
a



b

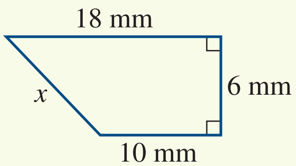


c



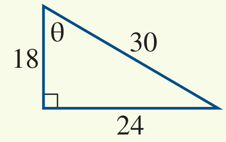
2 A rectangular block of land measures 20 m by 27 m. A fence is required along its diagonal. How long will the fence be? (Answer correct to one decimal place.)

3 Calculate the length of x, correct to the nearest millimetre.



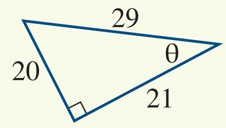
4 In the triangle shown, state the value of the:

a hypotenuse **b** opposite side **c** adjacent side.



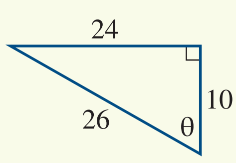
5 What are these ratios?

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



6 What are these ratios in simplest form?

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



7 Find the value of the following trigonometric ratios, correct to two decimal places.

a $\tan 68^\circ$ **b** $\cos 13^\circ$ **c** $\sin 23^\circ$ **d** $\cos 82^\circ$

8 Given the following trigonometric ratios, find the value of θ to the nearest degree.

a $\cos\theta = 0.4829$ **b** $\sin\theta = \dfrac{1}{3}$ **c** $\tan\theta = 0.2$

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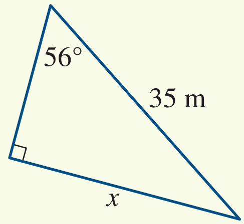
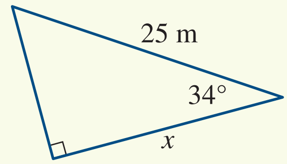
Review

9 Find the value of x, correct to two decimal places.

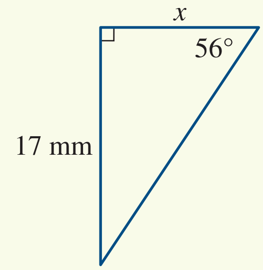
a



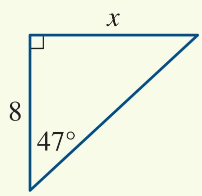
b



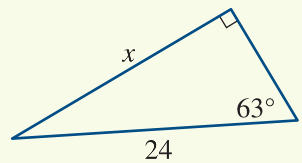
c



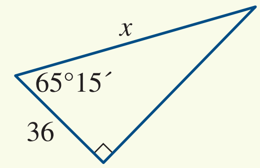
d



e

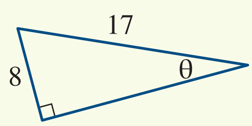


f



10 Find the unknown angle θ in each triangle. Answer correct to the nearest minute.

a



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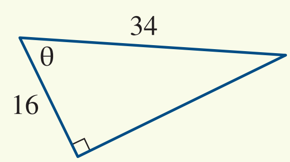

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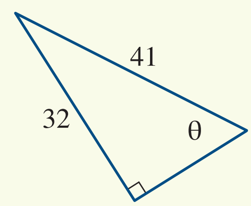

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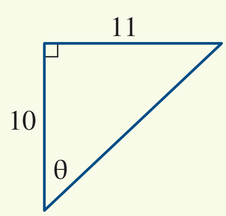

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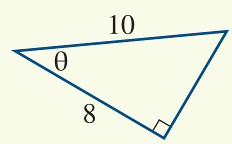

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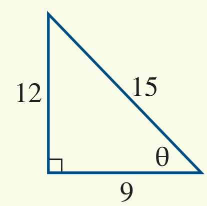
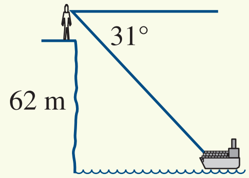


11 Susan looked from the top of a cliff, 62 m high, and noticed a ship at an angle of depression of 31°. How far was the ship from the base of the cliff? Answer correct to one decimal place.



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12 Emma rode for 8.5 km on a bearing of N43°W from her home.

a How far north is Emma from home? Answer correct to one decimal place.

b How far west is Emma from home? Answer correct to one decimal place.

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