5 Simultaneous linear equations

Syllabus topic — A3.1 Simultaneous linear equations

This topic will develop your understanding of the use of simultaneous linear equations in solving practical problems.

Outcomes

- Graph linear functions.
- Interpret linear functions as models of physical phenomena.
- Develop linear equations from descriptions of situations.
- Solve a pair of simultaneous linear equations using graphical methods.
- Finding the point of intersection between two straight-line graphs.
- Develop a pair of simultaneous linear equations to model a practical situation.
- Solve practical problems by modelling with a pair of simultaneous linear functions.
- Apply break-even analysis to solve simple problems.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

Mathematics Standard 1
Cambridge Maths Stage 6
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5A Linear functions

A linear function makes a straight line when graphed on a number plane. The linear function $y = 3x - 2$ has two variables y and x. When a number is substituted for a variable, such as $x = 2$, then this variable is called the independent variable. The dependent variable depends on the number substituted for the independent variable. That is, when $x = 2$ (independent) then $y = 3 \times 2 - 2$ or 4 (dependent).

To graph a linear function, construct a table of values with the independent variable as the first row and the dependent variable as the second row. Plot these points on the number plane with the independent variable on the horizontal axis and the dependent variable as the vertical axis. Join the points to make a straight line.

GRAPHING A LINEAR FUNCTION

1. Construct a table of values with the independent variable as the first row and the dependent variable as the second row.
2. Draw a number plane with the independent variable on the horizontal axis and the dependent variable as the vertical axis. Plot the points.
3. Join the points to make a straight line.

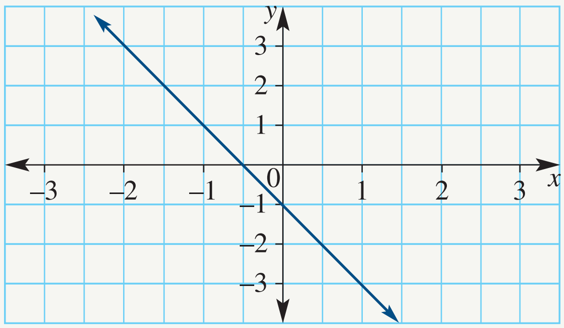
Example 1: Drawing a linear function 5A

Draw the graph of $y = -2x - 1$.

SOLUTION:

1. Draw a table of values for x and y.
2. Let $x = -2, -1, 0, 1$ and 2. Find y using the linear function $y = -2x - 1$.
3. Draw a number plane with x as the horizontal axis and y as the vertical axis.
4. Plot the points $(-2, 3)$, $(-1, 1)$, $(0, -1)$ and $(1, -3)$. The point $(2, -5)$ has not been plotted as it does not fit the scale of the number plane.
5. Join the points to make a straight line.

x	−2	−1	0	1	2
y	3	1	−1	−3	−5



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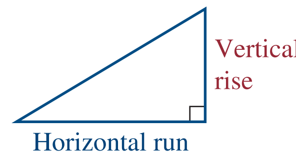
Gradient–intercept formula

When the equation of a straight line is written in the form $y = mx + c$ (or $y = mx + b$) it is called the gradient–intercept formula. The gradient is m or the coefficient of x. It is the slope or steepness of the line. The gradient of a line is calculated by dividing the vertical rise by the horizontal run. Lines that go up to the right (/) have positive gradients and lines that go down to the right (\) have negative gradients.



$$\text{Gradient (or } m) = \frac{\text{Vertical rise}}{\text{Horizontal run}}$$

The intercept of a line is where the line cuts the axis. The intercept on the vertical axis is called the y-intercept and is denoted by the letter c. (Previously in this course, b was used.)

GRADIENT–INTERCEPT FORMULA

Linear equation: $y = mx + c$.
m – Slope or gradient of the line (vertical rise over the horizontal run).
c – y-intercept. Where the line cuts the y-axis or vertical axis.

Example 2: Draw a graph from a table of values, find gradient and y-intercept 5A

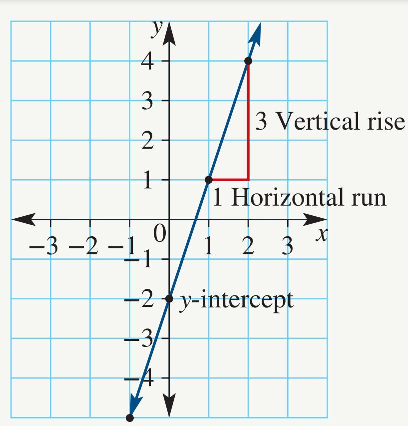
Draw the graph of $y = 3x - 2$ from a table of values. Find the gradient and y-intercept of this line, and check that they form a linear equation that is the same as the original one.

SOLUTION:

1 Construct a table of values for x and y.
2 Let $x = -2, -1, 0, 1$ and 2. Find y using the linear function $y = 3x - 2$.

x	−2	−1	0	1	2
y	−8	−5	−2	1	4

3 Draw a number plane with x as the horizontal axis and y as the vertical axis.
4 Plot the points $(-1, -5)$, $(0, -2)$, $(1, 1)$ and $(2, 4)$. The point $(-2, -8)$ has not been plotted as it does not fit the scale of the number plane.
5 Join the points to make a straight line. Calculate the gradient of the line using the 'rise over run formula' and read the value of the y-intecept where the line crosses the vertical axis.



6 Write the values of the gradient and y-intercept. — Gradient m is 3, y-intercept c is −2.
7 Write equation of the line in the form $y = mx + c$. The gradient is the coefficient of x and the y-intercept is −2. — $y = 3x - 2$
8 Compare this equation with the question. — The equations are the same.

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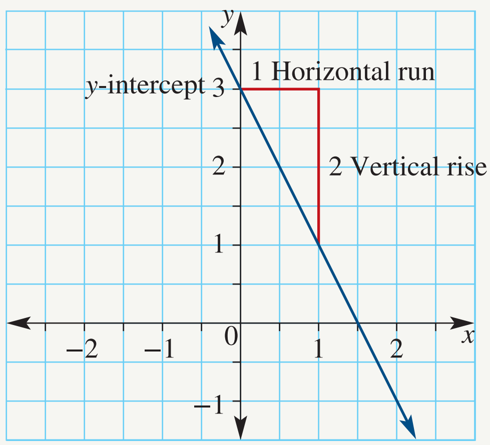
Sketching a straight line requires at least two points. When an equation is written in gradient–intercept form, one point on the graph is immediately available: the y-intercept. A second point can be quickly calculated using the gradient.

Example 3: Sketching a linear function using the gradient and y-intercept 5A

Sketch the graph of $3y + 6x = 9$.

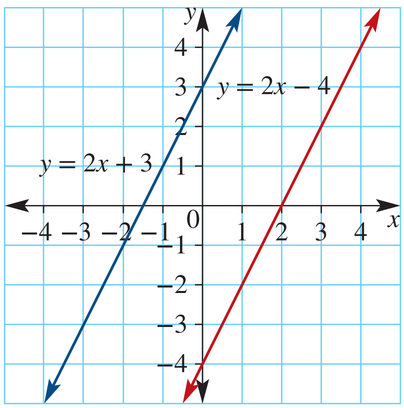
SOLUTION:

1. Rearrange the equation into gradient form $y = mx + c$. $\quad 3y + 6x = 9$
2. Subtract $6x$ from both sides. $\quad 3y = 9 - 6x$
3. Divide both sides by 3 and simplify. $\quad y = \dfrac{9 - 6x}{3}$, $\quad y = 3 - 2x$
4. The equation is now written in the form $y = mx + c$. $\quad y = -2x + 3$
5. Therefore $m = -2$ and $c = 3$. $\quad$ Gradient is -2 and y-intercept is 3
6. A gradient of -2 means that for every unit across in the positive x-axis direction, you go down 2 in the negative y-axis direction.
7. Plot the y-intercept (0, 3), then move across 1 (horizontal run) and down 2 (vertical rise) to plot the point (1, 1).
8. Join the points (0, 3) and (1, 1) to make a s traight line.



Parallel lines

Consider the linear function $y = 2x + 3$.
It has a gradient of 2 and y-intercept of 3.
Consider the linear function $y = 2x - 4$.
It has a gradient of 2 and y-intercept of -4.
The graph of these linear functions is shown opposite.
They are parallel because they both have the same gradient of $m = 2$.



PARALLEL LINES

If the value of m is the same for two linear functions, then the lines are parallel.

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Exercise 5A

1 Plot the following points on a number plane and join them to form a straight line.

a

x	-2	-1	0	1	2
y	2	1	0	-1	-2

b

x	-2	-1	0	1	2
y	-3	-1	1	3	5

2 Complete the following table of values and graph each linear function.

a $y = x - 1$

x	0	1	2	3	4
y					

b $y = -2x$

x	0	2	4	6	8
y					

c $y = 2x + 3$

x	-2	-1	0	1	2
y					

d $y = -x + 2$

x	-2	-1	0	1	2
y					

Example 1

3 Draw the graphs of these linear functions by first completing a table of values.

a $y = 2x + 2$

b $y = -x + 3$

c $y = \frac{2}{3}x - 1$

d $y = -\frac{1}{2}x + 1$

e $y = 3x - 1$

f $y = -2x + 3$

Example 2

4 Find the gradient of the following straight lines.

a $y = 5x + 1$

b $y = x - 2$

c $y = -2x$

d $y = \frac{1}{2}x + 4$

e $y = -\frac{2}{3}x + 3$

f $y = \frac{3}{4}x + \frac{1}{2}$

5 Find the y-intercept of the following straight lines.

a $y = 3x - 5$

b $y = -x + 2$

c $y = -4x$

d $y = -\frac{1}{3}x - 1$

e $y = -\frac{2}{5}x + 7$

f $y = \frac{1}{4}x + \frac{3}{5}$

6 Find the equation of the following straight lines defined by:

a gradient 2, passing through (0, 1)

b gradient −3, passing through (0, 4)

c gradient 0.5, passing through (0, −2)

d gradient 0, passing through (0, 6)

e gradient $-\frac{2}{5}$, passing through (0, 4)

f gradient $\frac{1}{3}$, passing through (0, 0).

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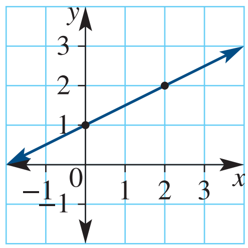
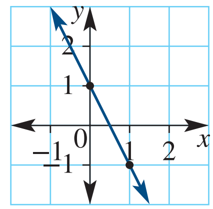
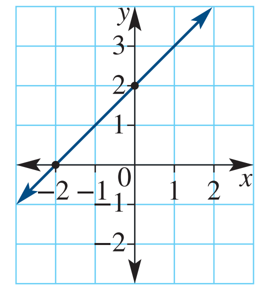
7 What is the equation of each of the following line graphs?

a


b


c


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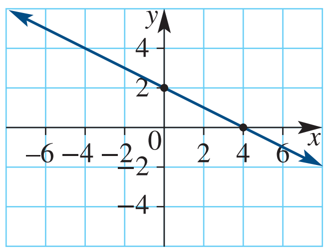

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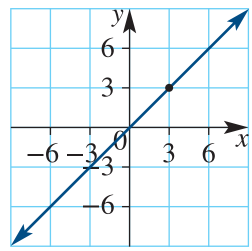

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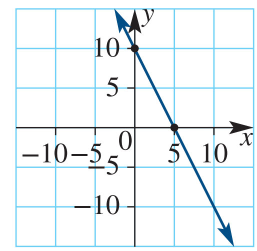
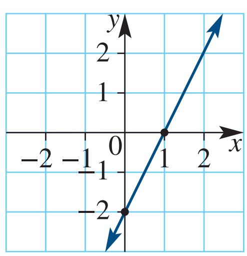


g


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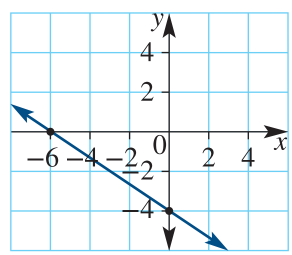
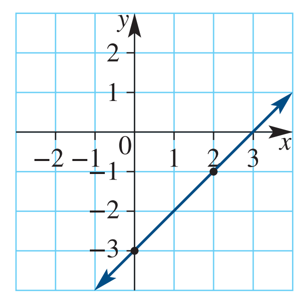


i


8 Which of the following lines are parallel?

a $y = 2x + 1$ and $y = x + 2$

b $y = -2x + 4$ and $y = 2x + 4$

c $y = 3x + 1$ and $y = 3x + 2$

d $y = -4x + 1$ and $y = -4x$

9 Find the equation of the line passing through each of the following pairs of points.

a (0, 3), (3, 0)

b (−2, 0), (0, 4)

c (2, 0), (0, 2)

d (1, 0), (0, −1)

e (−1, 0), (0, 3)

f (0, 4), (4, 2)

10 Express the following linear equations in gradient–intercept form ($y = mx + c$).

a $y + 2 = 3x$

b $x + y - 4 = 0$

c $y - \frac{1}{2}x = 1$

d $4x - y + 2 = 0$

e $\frac{1}{3}x - y = 1$

f $4 - y = 3x$

11 Draw the graph of the linear functions in question **10** using a table of values.

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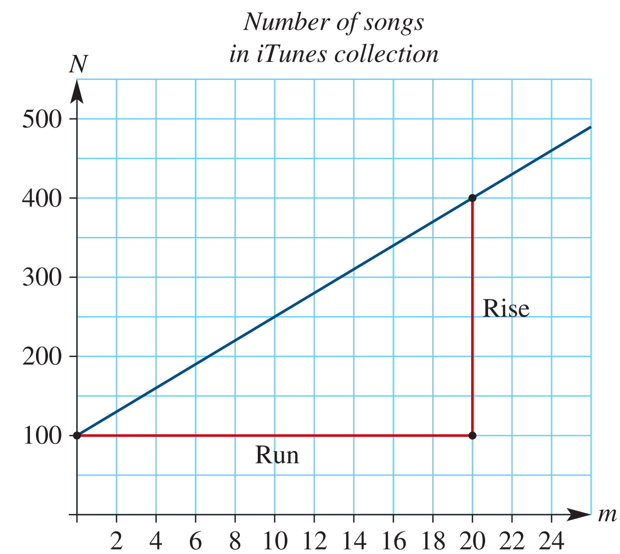
5B Linear models

Linear modelling occurs when a practical situation is described mathematically using a linear function. For example, the gradient–intercept form of a straight-line graph can be used to model an iTunes collection. Logan owns 100 songs in his iTunes collection and adds 15 new songs each month. Using this information, we can write a linear equation to model the number of songs in his collection. Letting N be the number of songs and m be the number of months, we can write $N = 15m + 100$.

Note: The number of months (m) must be greater than zero and a whole number.



The graph of this linear model has been drawn opposite. There are two important features of this linear model:

1. Gradient is the rate per month or $15\left(\frac{300}{20}\right)$ songs.
2. The vertical axis intercept is the initial number of songs or 100.

LINEAR MODELS

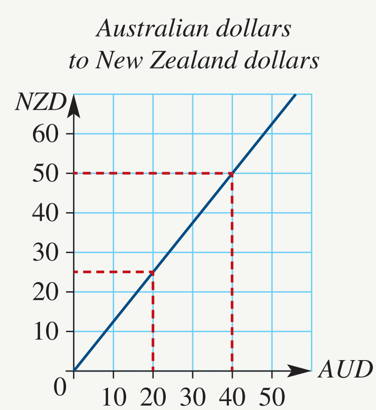
Linear models describe a practical situation mathematically using a linear function.

Example 4: Using linear models — 5B

The graph opposite is used to convert Australian dollars (AUD) to New Zealand dollars (NZD). Use the graph to convert:

a 40 AUD to NZD

b 25 NZD to AUD.



SOLUTION:

1 Read from the graph (when AUD = 40, NZD = 50).	**a** 50 NZD
2 Read from the graph (when NZD = 25, AUD = 20).	**b** 20 AUD

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Example 5: Interpreting linear models 5B

Grace sells insurance. She earns a base salary and a commission on each new insurance policy she sells. The graph shows Grace's weekly income (I) plotted against the number of new policies (n) she sells in that week. The relationship between I and n is linear.

a What is Grace's base salary?

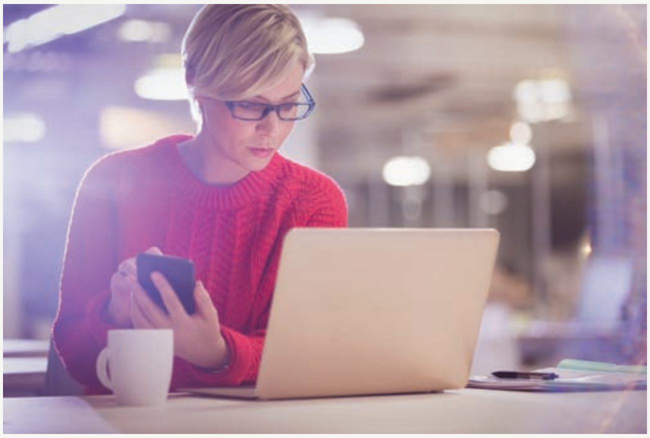
b What is Grace's salary in a week in which she sells 8 new policies?

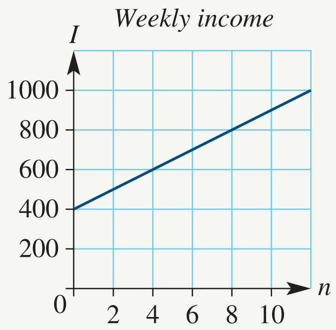
c How many policies does Grace need to sell to earn \$700 for the week?

d Find the equation of the straight line in terms of I and n.

e Use the equation to calculate the weekly income when Grace sells 3 new policies.

f How much does Grace earn for each new policy?





SOLUTION:

1 Read from the graph (when $n = 0$, $I = 400$).	**a** \$400
2 Read from the graph (when $n = 8$, $I = 800$).	**b** \$800
3 Read from the graph (when $I = 700$, $n = 6$).	**c** 6 new policies
4 Find the gradient by choosing two suitable points. (0, 400) and (8, 800).	**d** $m = \frac{\text{Rise}}{\text{Run}}$, $c = 400$
5 Calculate the gradient (m) between these points using the gradient formula.	$= \frac{800 - 400}{8 - 0}$
6 Determine the vertical intercept (400).	$= 50$
7 Substitute the gradient and y-intercept into the gradient–intercept form $y = mx + c$.	$y = mx + c$ $I = 50n + 400$
8 Use the appropriate variables (I for y, n for x).	**e** $I = 50n + 400$
9 Substitute $n = 3$ into the equation.	$= 50 \times 3 + 400$
10 Evaluate.	$= \$550$
11 Check the answer using the graph.	
12 The gradient of the graph is the commission for each new policy.	**f** \$50

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Exercise 5B

1 Complete the following tables of values and graph each linear function.

a $c = d + 5$

c	0	1	2	3	4
d					

b $I = -3n$

I	0	2	4	6	8
n					

2 Draw the graph of these linear functions using a table of values from 1 to 10.

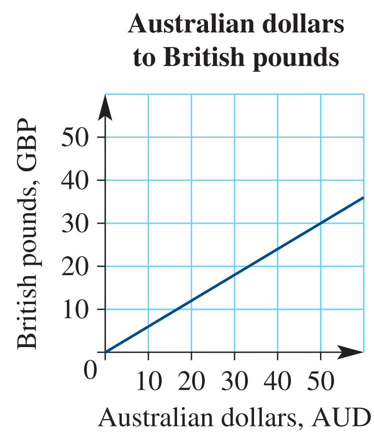
a $C = 2n + 10$ **b** $V = -5t + 30$ **c** $M = \frac{1}{2}n - 10$

Example 4

3 **a** The conversion graph opposite is used to convert Australian dollars to British pounds. Use the graph to calculate these exchanges.

i 30 Australian dollars to pounds
ii 50 Australian dollars to pounds
iii 20 pounds to Australian dollars
iv 10 pounds to Australian dollars

b What is the gradient of the conversion graph?



4 The relationship of the age of machinery (a) in years to its value (v) in \$1000 is $v = -4a + 20$.

a Construct a table of values for age against value. Use values of a from 0 to 4.
b Draw the graph of age (a) against value (v).
c What is the initial cost of the machinery?
d What is the age of the machinery if its current value is \$15 000?
e What is the value of the machinery after $3\frac{1}{2}$ years?
f What will be the value of the machinery after 2 years?
g When will the machinery be worth half its initial cost?

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Example 5 **5** An ultrasound machine was purchased by a medical centre for $150000. Its value is depreciated each month as shown in the graph.

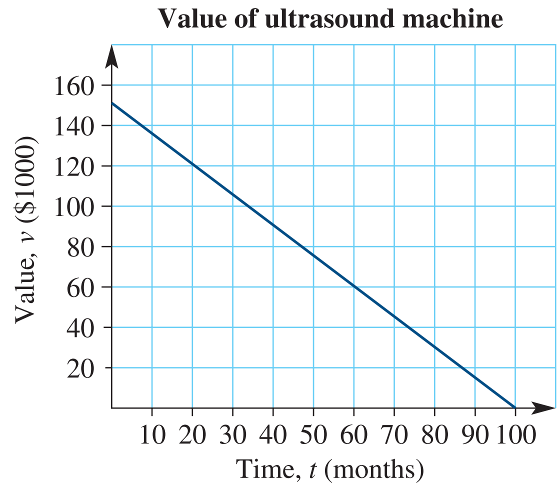
a What was the value of the machine after 40 months?

b What was the value of the machine after five years?

c When does the line predict the machine will have no value?

d Find the equation of the straight line in terms of v and t.

e Use the equation to predict the value of the machine after 6 months.



6 A car is travelling at constant speed. It travels 360 km in 9 hours.

a Write a linear equation in the form $d = mt$ to describe this situation.

b Draw the graph of d against t.

7 The cost ($$C$) of hiring a taxi consists of two elements: a fixed flagfall and a figure that varies with the number (n) of kilometres travelled. If the flagfall is $2.60 and the cost per kilometre is $1.50, determine a rule that gives C in terms of n.

8 The weekly wage, $$w$, of a vacuum cleaner salesperson consists of a fixed sum of $350 plus $20 for each cleaner sold. If n cleaners are sold per week, construct a rule that describes the weekly wage of the salesperson.

9 A telecommunications company's rates for local calls from private telephones consist of a quarterly rental fee of $40 plus 25$c$ for every call. Construct a linear rule that describes the quarterly telephone bill. Let C be the cost (in cents) of the quarterly telephone bill and n the number of calls.

10 Blake converted 100 Australian dollars (AUD) to 60 euros (EUR).

a Draw a conversion graph with Australian dollars on the horizontal axis and euros on the vertical axis.

b How many euros is 25 Australian dollars? Use the conversion graph.

c How many Australian dollars is 45 euros? Use the conversion graph.

d Find the gradient and vertical intercept for the conversion graph.

e Write an equation that relates Australian dollars (AUD) to euros (EUR).

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5C Simultaneous equations – graphically

Two straight lines will always intersect unless they are parallel. The point at which two straight lines intersect can be found by sketching the two graphs on the one set of axes and reading off the coordinates of the point of intersection. Finding the point of intersection is said to be 'solving the equations simultaneously'. In addition to graphing the straight lines, the point of intersection could be determined by looking at the table of values. If the same value for x and y occurs in both tables it is the point of intersection. See example below.

SOLVING A PAIR OF SIMULTANEOUS EQUATIONS GRAPHICALLY

1. Draw a number plane.
2. Graph both linear equations on the number plane.
3. Read the point of intersection of the two straight lines.
4. Interpret the point of intersection for practical applications.

Example 6: Finding the solution of simultaneous linear equations — 5C

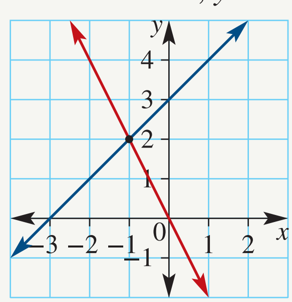
Find the simultaneous solution of $y = x + 3$ and $y = -2x$.

SOLUTION:

1. Use the gradient–intercept form to determine the gradient (coefficient of x) and y-intercept (constant term) for each line.

$y = x + 3$
Gradient is $+1$, y-intercept is 3.
$y = -2x$
Gradient is -2, y-intercept is 0.

2. Draw a number plane.
3. Sketch $y = x + 3$ using the y-intercept of 3 and a gradient of 1.
4. Sketch $y = -2x$ using the y-intercept of 0 and a gradient of -2.



5. Find the point of intersection of the two lines.

$(-1, 2)$

6. The simultaneous solution is the point of intersection.

Simultaneous solution is $x = -1$ and $y = 2$, $(-1, 2)$.

7. Alternatively, construct a table of values for x and y. Let $x = -2, -1, 0, 1$ and 2. Find y using the linear function $y = x + 3$.

x	−2	−1	0	1	2
y	1	2	3	4	5

8. Repeat to find y using the linear function $y = -2x$.

x	−2	−1	0	1	2
y	4	2	0	−2	−4

9. The same value of x and y occurs in both tables when $x = -1$ and $y = 2$.

Simultaneous solution is $x = -1$ and $y = 2$.

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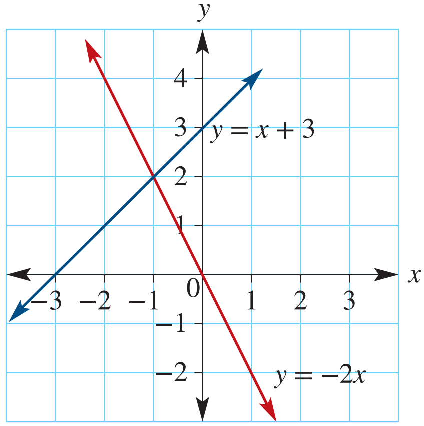
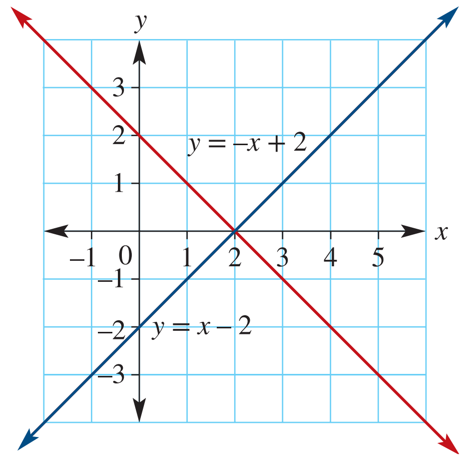
Exercise 5C

1 What is the point of intersection for each of these pairs of straight lines?

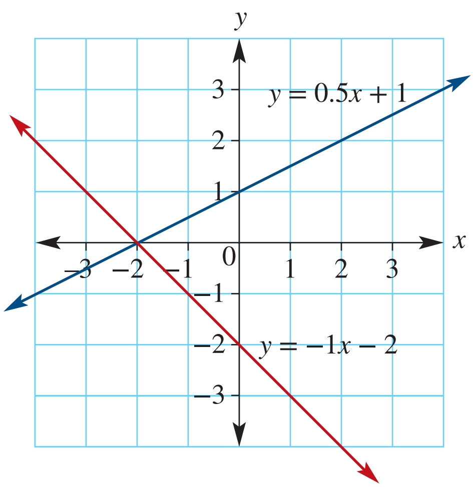
a



b



c



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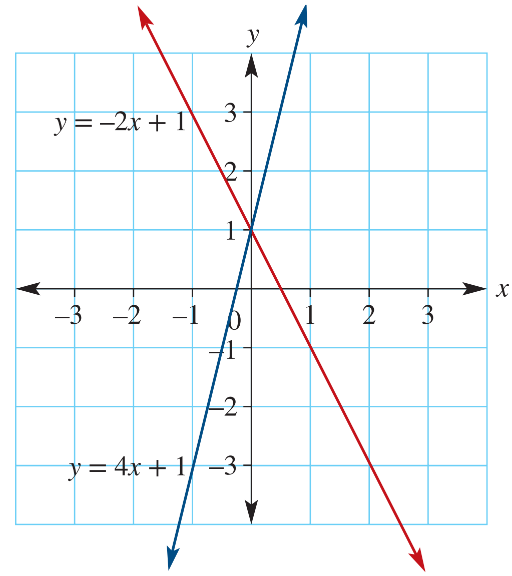
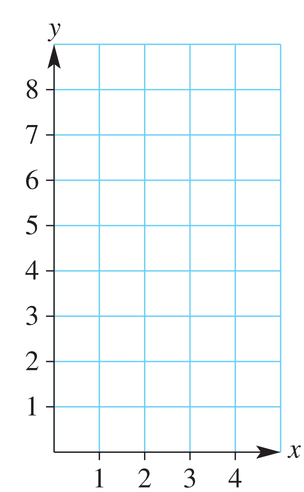


Example 6 **2** Plot the following points on a number plane and join them to form two straight lines. What is the point of intersection of these straight lines?

x	0	1	2	3	4
y	0	2	4	6	8

x	0	1	2	3	4
y	6	5	4	3	2



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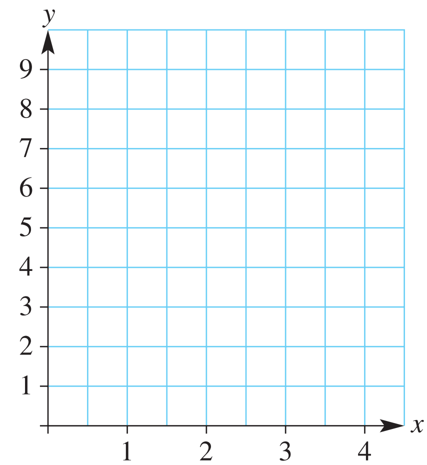
3 Plot the following points on a number plane and join them to form two straight lines. What is the point of intersection of these straight lines?

x	0	1	2	3	4
y	1	3	5	7	9

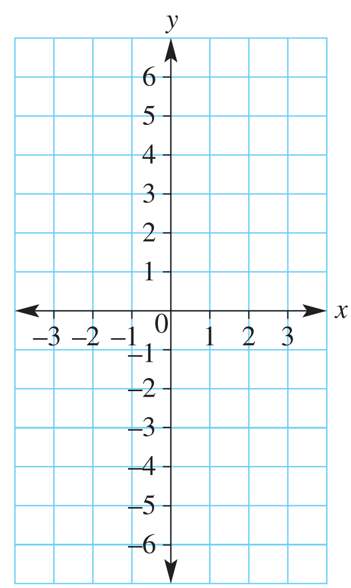
x	0	1	2	3	4
y	4	3	2	1	0



4 Plot the following points on a number plane and join them to form two straight lines. What is the point of intersection of these straight lines?

x	−2	−1	0	1	2
y	−6	−5	−4	−3	−2

x	−2	−1	0	1	2
y	6	3	0	−3	−6



5 Complete the following table of values and plot the points on a number plane. Find the simultaneous solution of these pairs of equations. All solutions are whole numbers.

a $y = 2x + 3$

x	−2	−1	0	1	2
y					

$y = -x$

x	−2	−1	0	1	2
y					

b $y = x + 4$

x	−2	−1	0	1	2
y					

$y = 2x$

x	−2	−1	0	1	2
y					

c $y = 3x + 1$

x	−2	−1	0	1	2
y					

$y = 5x - 3$

x	−2	−1	0	1	2
y					

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6 Complete the following table of values and plot the points on a number plane. What is the solution to each pair of simultaneous equations? Some of the solutions are not whole numbers.

a $y = 3x - 3$

x	−2	−1	0	1	2
y					

$y = x + 1$

x	−2	−1	0	1	2
y					

b $y = 3x - 2$

x	−2	−1	0	1	2
y					

$y = -x + 4$

x	−2	−1	0	1	2
y					

c $y = x$

x	−2	−1	0	1	2
y					

$y = 4x + 3$

x	−2	−1	0	1	2
y					

d $y = -x$

x	−6	−3	0	3	6
y					

$y = 4 - 2x$

x	−6	−3	0	3	6
y					

e $y = 5x + 1$

x	−6	−4	−2	0	2
y					

$y = 3x - 7$

x	−6	−4	−2	0	2
y					

f $y = x + 1$

x	−2	−1	0	1	2
y					

$y = -2x$

x	−2	−1	0	1	2
y					

g $y = 2x - 4$

x	0	1	2	3	4
y					

$y = -x + 5$

x	0	1	2	3	4
y					

h $y = x + 1$

x	−2	−1	0	1	2
y					

$y = -3x + 2$

x	−2	−1	0	1	2
y					

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5D Simultaneous equation models

When two practical situations are described mathematically using a linear function then the point of intersection has an important and often different meaning depending on the situation. For example, when income is graphed against costs the point of intersection represents the point where a business changes from a loss to a profit.

SIMULTANEOUS EQUATIONS AS MODELS

Simultaneous equation models use two linear functions to describe a practical situation and the point of intersection is often the solution to a problem.

Example 7: Using simultaneous equations as models 5D

Zaina buys and sells books. Income received by selling a book is calculated using the formula $I = 16n$. Costs associated in selling a book are calculated using the formula $C = 8n + 24$.

a What is the income when 6 books are sold?
b What is the costs when 6 books are sold?
c Draw the graph of $I = 16n$ and $C = 8n + 24$ on same number plane.
d Use the graph to determine the number of books needed to be sold for the costs to equal the income.

SOLUTION:

1 Substitute 6 for n into the formula for income $I = 16n$.

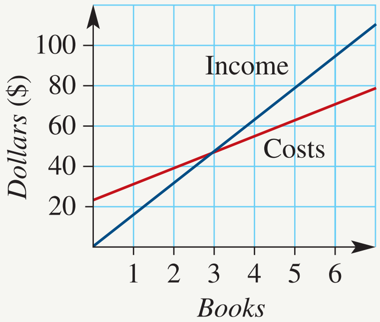
a $I = 16n = 16 \times 6 = 96$
$\therefore$ Income for six books is $96

2 Substitute 6 for n into the formula for costs $C = 8n + 24$.

b $C = 8n + 24 = 8 \times 6 + 24 = \$72$
$\therefore$ Costs for six books is $72

3 Draw a number plane.
4 Use the gradient–intercept form to determine the gradient and vertical intercept for each line. Gradient is the coefficient of n. Vertical intercept is the constant term.
5 Sketch $I = 16n$ using the vertical intercept of 0 and gradient of 16.
6 Sketch $C = 8n + 24$ using the vertical intercept of 24 and gradient of 8.

c



7 Find the point of intersection of the two lines (3, 48).

d Income is equal to costs when $n = 3$
$\therefore$ 3 books

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Example 8: Solving problems using intersecting graphs 5D

Isabella's Mathematics mark exceeded her English mark by 15. She scored a total of 145 for both tests. Find Isabella's marks in each subject by plotting intersecting graphs.

SOLUTION:

1 Express the relationship between the Mathematics and the English mark as a linear equation.

Let the Mathematics mark be m.
Let the English mark be e.
$m = e + 15$

2 Use the gradient–intercept form to determine the gradient and vertical intercept for the line. Gradient is the coefficient of e. Vertical intercept is the constant term.

Gradient is 1,
vertical intercept is 15.

3 Express the total of the two marks as a linear equation.

$m + e = 145$

4 Use the gradient–intercept form to determine the gradient and vertical intercept for the line.

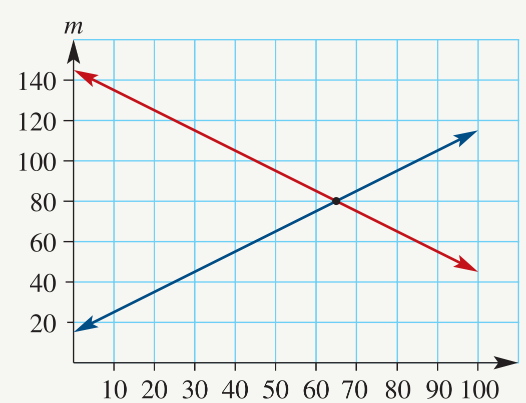
$m = -e + 145$
Gradient is -1,
vertical intercept is 145.

5 Draw a number plane.

6 Sketch $m = e + 15$ using the vertical intercept of 15 and gradient of 1.

7 Sketch $m = -e + 145$ using the vertical intercept of 145 and a gradient of -1.

8 The simultaneous solution is the point of intersection.



9 Find the point of intersection of the two lines.

Intersection is (65, 80) so $e = 65$ and $m = 80$

10 Write the solution in words using the context of the question.

Isabella scored 65 in English and 80 in Mathematics.

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Exercise 5D

1 Matilda and Nathan earn wages m and n respectively.

a Matilda earns \$100 more than Nathan. Write an equation to describe this information.

b The total of Matilda's wages and Nathan's wages is \$1200. Write an equation to describe this information.

c Draw a graph of the two equations on the same number plane. Use n as the horizontal axis and m as the vertical axis.

d Use the intersection of the two graphs to find Matilda's and Nathan's wages.

2 Let one number be represented by a and the other number by b.

a The sum of the two numbers is 42. Write an equation to describe this information.

b The difference of the two numbers is 6. Write an equation to describe this information.

c Draw a graph of the two equations on the same number plane. Use a as the horizontal axis and b as the vertical axis.

d Use the intersection of the two graphs to find the two numbers.

3 Let one number be represented by p and another number by q.

a The sum of the two numbers is 15. Write an equation to describe this information.

b One of the numbers is twice the other number. Write an equation to describe this information.

c Draw a graph of the two equations on the same number plane. Use p as the horizontal axis and q as the vertical axis.

d Use the intersection of the two graphs to find the two numbers.

4 Amy and Nghi work for the same company and their wages are a and b respectively.

a Amy earns \$100 more than Nghi. Write an equation to describe this information.

b The total of Amy's and Nghi's wages is \$1500. Write an equation to describe this information.

c Draw a graph of the above two equations on the same number plane. Use a as the horizontal axis and b as the vertical axis.

d Use the intersection of the two graphs to find Amy's and Nghi's wages.

Example 7, 8

5 A factory produces items whose costs are \$1000 plus \$10 for every item. The factory receives \$60 for every item sold.

a Write an equation to describe the relationship between the:

i costs (C) and number of items (n)

ii income (I) and number of items (n).

b Draw a graph and find the number of items when income equals costs.

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5E Break-even analysis

The break-even point is reached when costs or expenses and income are equal. There is no profit or loss at the break-even point. For example, if the break-even point for a business is 100 items per month, the business will make a loss if it sells fewer than 100 items each month; if it sells more than 100 items per month, it will make a profit. A profit (or loss) is calculated by subtracting the costs from the income (Profit = Income – Costs). Income is a linear function of the form $I = mx$, where x is the number of items sold and m is the selling price of each item. Cost is a linear function of the form $C = mx + c$, where x is the number of items sold, m is the cost price per item manufactured and c is the fixed costs of production.

BREAK-EVEN ANALYSIS

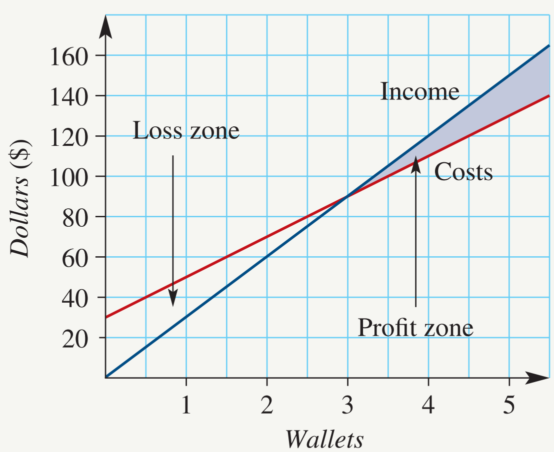
Break-even point occurs when costs equal income.
Profit = Income − Costs
Income: $I = mx$
Costs: $C = mx + c$

Example 9: Interpreting the point of intersection of two graphs 5E

Grace buys and sells wallets. Income received by selling wallets is calculated using the formula $I = 30x$. Costs associated with selling wallets are calculated using the formula $C = 20x + 30$.

a Use the graph to determine the number of wallets that Grace needs to sell to break even.

b How much profit or loss does she make when four wallets are sold?



SOLUTION:

1 Consider when the break-even point occurs.	**a** When the income equals the costs.
2 Read the point of intersection of the two linear graphs.	Intersection is at (3, 90). So $x = 3$. Number of wallets $= 3$
3 Profit is determined by subtracting the costs from the income.	**b** Profit = Income − Cost
4 Read from the graph the values of I and C when $x = 4$.	$I = 120$ and $C = 110$
5 Evaluate.	$= 120 - 110$ $= \$10$
6 Write the answer in words.	Profit for selling 4 wallets is $10.

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Example 10: Break-even analysis 5E

A firm sells its product at \$20 per unit. The cost of production (\$$C$) is given by the rule $C = 4x + 48$, where x is the number of units produced.

a Find the value of x for which the cost of the production of x units is equal to the income or revenue received by the firm for selling x units.

b Check your answer algebraically.

SOLUTION:

1 Set up the income equation and determine the gradient and vertical intercept.

a Let the income or revenue for producing x units be \$$I$. Formula is:

$$I = 20x$$

Gradient is 20, vertical intercept is 0

2 Set up the cost of production equation and determine the gradient and vertical intercept.

Cost of production (\$$C$) is given by:

$$C = 4x + 48$$

Gradient is 4, vertical intercept is 48

3 Draw a number plane.

4 Use x as the horizontal axis.

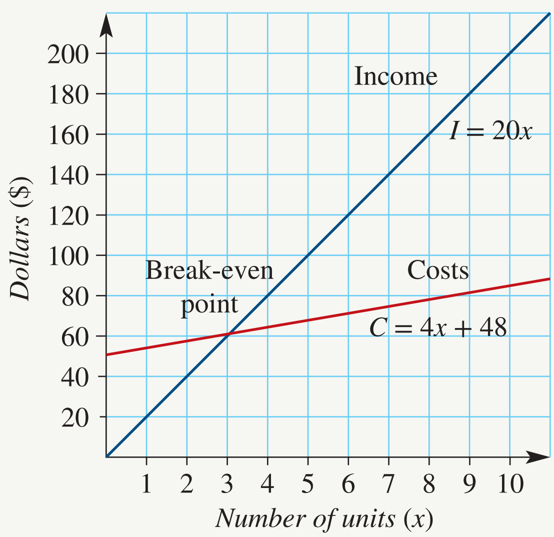
5 Use I and C as the vertical axis.

6 Sketch $I = 20x$ using the vertical intercept of 0 and gradient of 20.

7 Check this line using some valid points such as (1, 20).

8 Sketch $C = 4x + 48$ using the vertical intercept of 48 and gradient of 4.

9 Check this line using some valid points such as (1, 52).



10 Read the value of x at the point of intersection of the two linear graphs.

The point of intersection of the two linear graphs occurs when $x = 3$. This is the break-even point, the value of x for which cost of production is equal to income.

11 Substitute $x = 3$ into the formula $I = 20x$.

b Check algebraically.

12 Substitute $x = 3$ into the formula $C = 4x + 48$.

Income	Costs
$I = 20x$	$C = 4x + 48$
$= 20 \times 3$	$= 4 \times 3 + 48$
$= 60$	$= 60$

13 Check that I is equal to C.

Income equals costs, so answer to **a** is correct.

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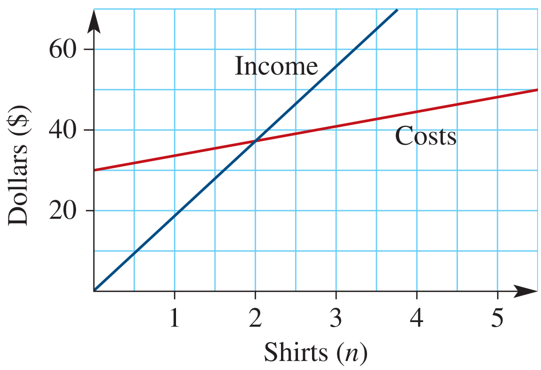
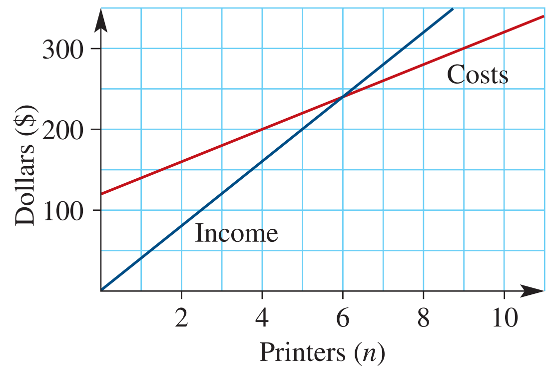
Exercise 5E

Example 9,10 **1** What is the break-even point for the following graphs?

a



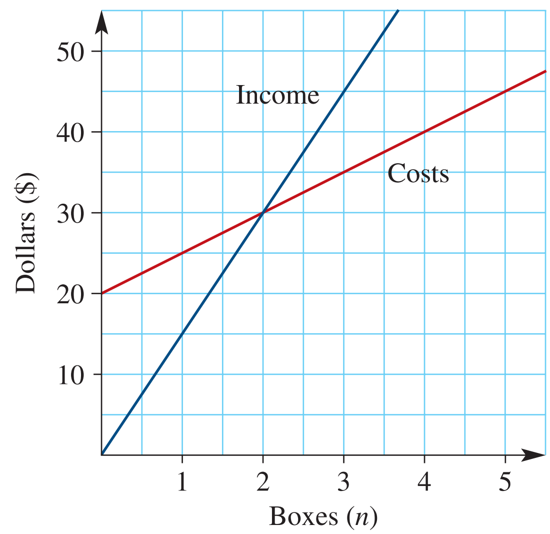
b



2 The graph on the right shows the cost of producing boxes of chocolates and the income received from their sale.

- **a** Use the graph to determine the number of boxes that need to be sold to break even.
- **b** How much profit or loss is made when 3 boxes are sold?
- **c** How much profit or loss is made when one box is sold?
- **d** What are the initial costs?

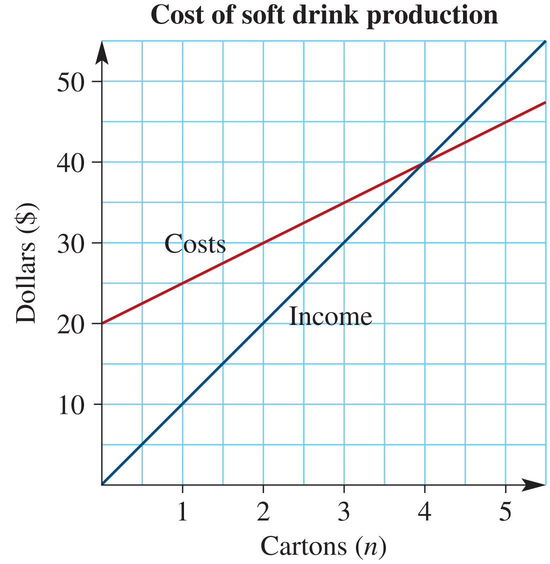
Cost of chocolate production



3 The graph below shows the cost of producing cartons of soft drinks and the income received from their sale.

- **a** Use the graph to determine the number of cartons that need to be sold to break even.
- **b** How much profit or loss is made when five cartons are sold?
- **c** How much profit or loss is made when two cartons are sold?
- **d** What is the initial cost?
- **e** What is the gradient of the straight line that represents income?
- **f** What is the vertical intercept of the straight line that represents income?
- **g** Write an equation to describe the relationship between income and the number of cartons.
- **h** What is the gradient of the straight line that represents costs?
- **i** What is the vertical intercept of the straight line that represents costs?
- **j** Write an equation to describe the relationship between costs and the number of cartons.

Cost of soft drink production



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PPT STUDY GUIDE

Summary

Key ideas and chapter summary

Linear functions

1 Construct a table of values with the independent variable as the first row and the dependent variable as the second row.
2 Draw a number plane with the independent variable on the horizontal axis and the dependent variable as the vertical axis. Plot the points.
3 Join the points to make a straight line.

Gradient–intercept formula

Linear equation: $y = mx + c$.

m – Slope or gradient of the line (vertical rise over the horizontal run).

c – y-intercept
Where the line cuts the y-axis or vertical axis.

Linear models

Linear models describe a practical situation mathematically using a linear function.

Simultaneous equations – graphically

1 Draw a number plane.
2 Graph both linear equations on the number plane.
3 Read the point of intersection of the two straight lines.
4 Interpret the point of intersection for practical applications (break-even point).

Simultaneous equation as models

Simultaneous equation models use two linear functions to describe a practical situation and the point of intersection is often the solution to a problem.

Break-even analysis

Break-even point occurs when costs equal income.

Profit = Income − Costs

Income: $I = mx$

Costs: $C = mx + c$

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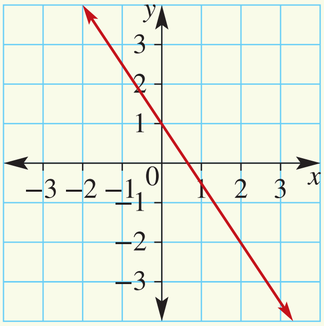
Review

Multiple-choice



1 What is the gradient of this line?

A $-\frac{3}{2}$ **B** $-\frac{2}{3}$ **C** $\frac{2}{3}$ **D** $\frac{3}{2}$

2 What is the y-intercept of this line?

A −2 **B** −1 **C** 1 **D** 2

3 A straight line has the equation of $y = -x - 3$. What is the y-intercept?

A −3 **B** −1 **C** +1 **D** +3

4 The cost of manufacturing bags (C) is given by the formula $c = 40x + 150$, where x is the number of bags sold. What is the cost of manufacturing two bags?

A \$40 **B** \$150 **C** \$190 **D** \$230

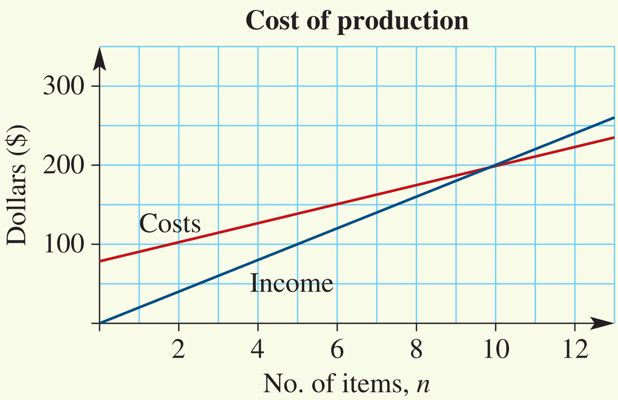
5 A car is travelling at a constant speed. It travels 80 km in 4 hours. This situation is described by the linear equation $d = mt$. What is the value of m?

A 0.05 **B** 3 **C** 20 **D** 60

6 What is the point of intersection of the lines $y = x + 2$ and $y = -x + 2$?

A (2, 0) **B** (0, 2) **C** (0, −2) **D** (1, 1)

Use the graph below to answer questions **7–9**.



7 What is the profit for selling 12 items?

A \$10 **B** \$20 **C** \$220 **D** \$240

8 What is the break-even point?

A 10 items **B** 12 items **C** 20 items **D** 80 items

9 What is the loss for selling 5 items?

A \$20 **B** \$30 **C** \$40 **D** \$50

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Review

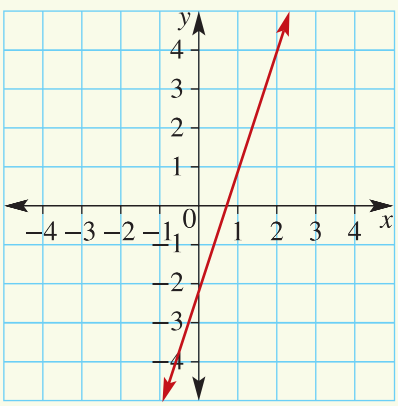
Short-answer

1 Draw the graph of these linear functions.

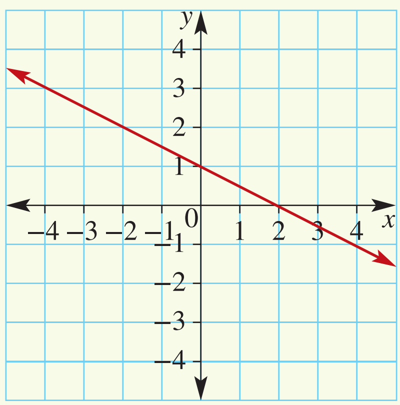
a $y = x + 2$ **b** $y = -3x + 1$ **c** $y = 2x - 2$

2 Find the equation of the following straight-line graphs.

a



b



3 The table below shows the speed v (in m/s) of a plane at time t seconds.

Time (t)	1	2	3	4	5
Speed (v)	2.5	4	5.5	7	8.5

a Draw a number plane with t as the horizontal axis and v as the vertical axis. Plot the points and join them to make a straight line.
b Determine a linear model in the form $y = mx + c$ to describe this situation.
c What does the model predict will be the plane's speed when $t = 2.5$ seconds?
d What does the model predict will be the plane's speed when $t = 6$ seconds?
e What does the model predict will be the plane's speed when $t = 7$ seconds?
f What does the model predict will be the plane's speed when $t = 10$ seconds?

4 An internet access plan charges an excess fee of $12 per GB.

Data (d)	1	2	3	4	5	6
Cost (c)	12	24	36	48	60	72

a Draw a graph of data against cost.
b Use the graph to find d if c is 30.
c Use the graph to find c if d is 3.5.
d Estimate the cost of 7 GB of data.
e Estimate the cost of 10 GB of data.
f Estimate the cost of 8.5 GB of data.

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Review

5 What is the point of intersection of the lines $y = 3x + 3$ and $y = x - 2$?

6 The graph opposite shows the cost of making picture frames and the income received from their sale.

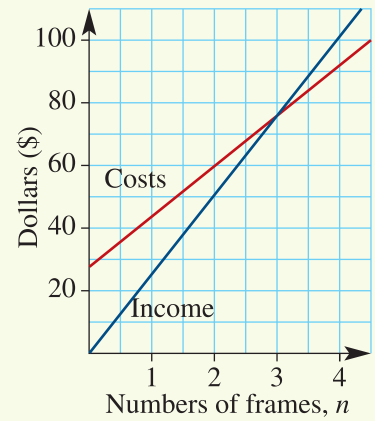
a Use the graph to determine the number of picture frames that need to be sold to break even.

b How much profit or loss is made when one picture frame is sold?

c How much profit or loss is made when four picture frames are sold?

d What is the initial cost?

Cost of picture frame manufacture



7 The graph opposite shows the cost of producing packs of batteries and the income received from their sale.

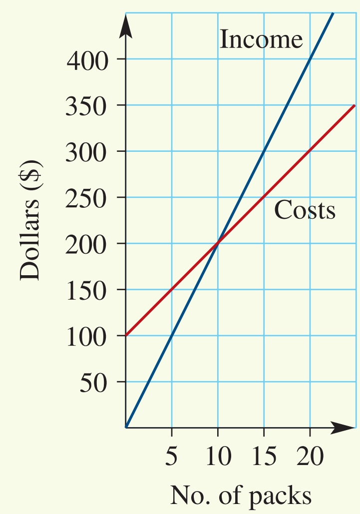
a Use the graph to determine the number of packs that need to be sold to break even.

b How much profit or loss is made when 5 packs are sold?

c How much profit or loss is made when 20 packs are sold?

d What are the initial costs?

Cost of battery production



8 The graph opposite shows the cost of manufacturing tables and the income received from their sale.

a Use the graph to determine the number of tables that need to be sold to break even.

b Write an equation to describe the relationship between income and the number of tables.

c Write an equation to describe the relationship between costs and the number of tables.

d How much profit or loss is made when 10 tables are sold?

e How much profit or loss is made when 4 tables are sold?

f How much profit or loss is made when 16 tables are sold?Simultaneous equations as models

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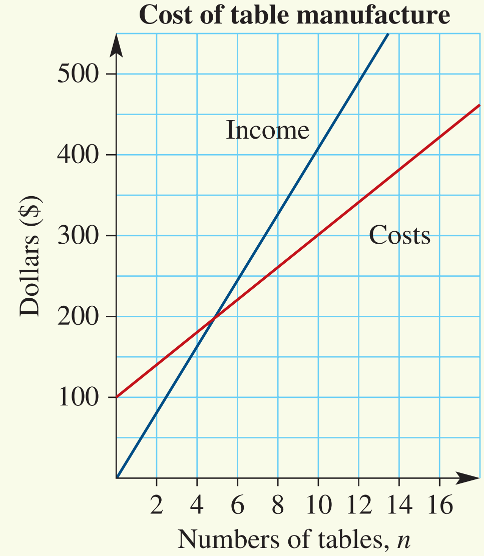
Cost of table manufacture



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