STD 1: Measurement (Std 1), M1 Applications of Measurement (Y11)

Trapezoidal Rule (Std 1)

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Exam Equivalent Time: 55.5 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

IMPORTANT FEATURES AND TIPS FROM 2UG EXAM HISTORY

- *MS-M1 Trapezoidal Rule* becomes the main approximation method for areas and volumes in the new Standard course, with *Simpson's Rule* no longer in the syllabus.
- Approximation methods accounted for an average of 2.1% of the old Gen2 HSC exam, and we expect a similar allocation in the Standard 1 exam going forward.
- Past HSC Simpson's Rule exam questions, where possible, have been rewritten into Trapezoidal Rule questions, adapting these high quality revision resources for the new course.

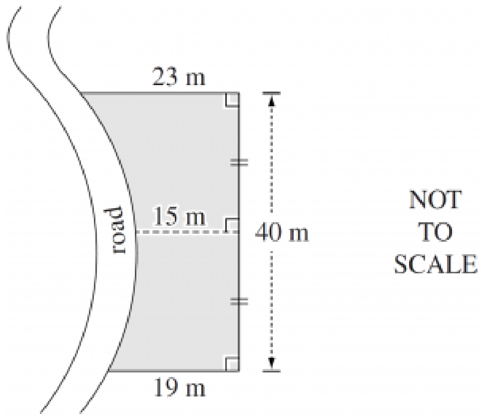
ANALYSIS - Common pitfalls

- Approximation methods in the old Gen2 course were tested in around half of the past exams, in questions with mark allocations ranging from 1-5 marks.
- Worth noting that the last two times this area has been examined, *volume* approximations were required (vs area approximations) - a slight variation that should be conceptually well understood.
- Important: while calculations associated with the Trapezoidal Rule itself are band 3 difficulty level, the "associated" cross topic questions have caused *major* difficulties in the past. A key feature of these difficulties concern unit conversion such as $km^2 \rightarrow m^2$ (see *Q.M1 SM-Bank 21*, and *Q.M1 SM-Bank 18*).

Questions

1. Measurement, STD2 M1 SM-Bank 15 MC

The shaded region represents a block of land bounded on one side by a road.



What is the approximate area of the block of land, using the Trapeziodal rule?

A. $720\ m^2$

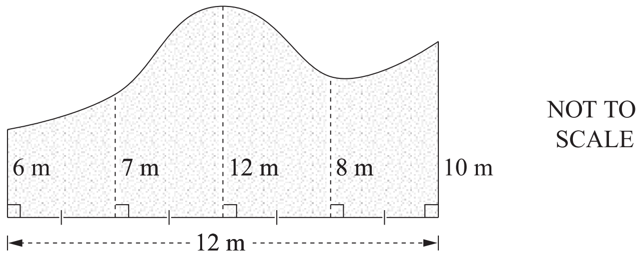
B. $880\ m^2$

C. $1140\ m^2$

D. $1440\ m^2$

2. Measurement, STD2 M1 SM-Bank 16 MC

The diagram represents a field.



What is the area of the field, using four applications of the Trapezoidal's rule?

A. 105 m²

B. 136 m²

C. 210 m²

D. 420 m²

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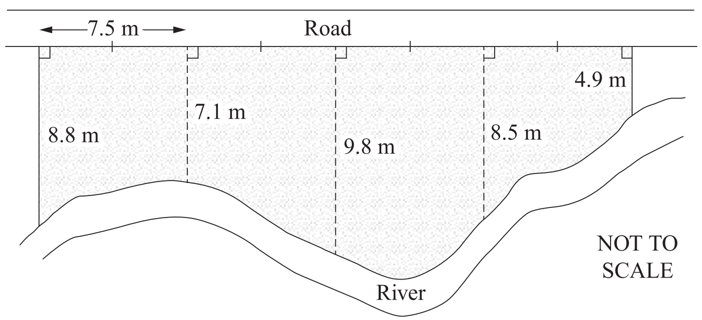
3. Measurement, STD2 2018 HSC 28a

A field is bordered on one side by a straight road and on the other side by a river, as shown. Measurements are taken perpendicular to the road every 7.5 metres along the road.



Use four applications of the Trapeziodal rule to find an approximation to the area of the field. Answer to the nearest square metre. *(3 marks)*

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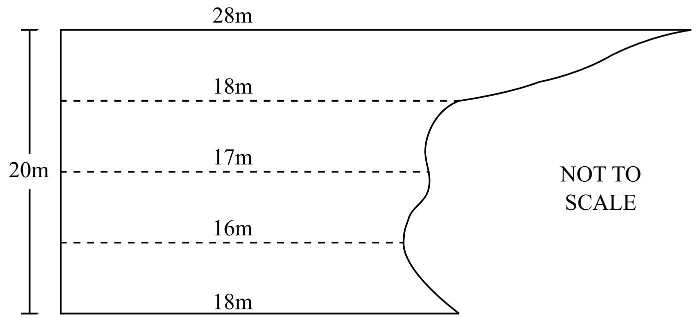
4. Measurement, STD2 M1 SM-Bank 02

A farmer wants to estimate the area of an irregular shaped paddock.



What is the estimated area of the land using the Trapezoidal Rule? *(2 marks)*

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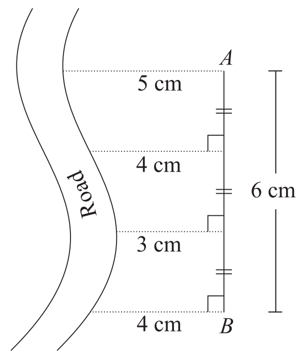
5. Measurement, STD2 M1 SM-Bank 12

The scale diagram shows the aerial view of a block of land bounded on one side by a road. The length of the block, AB, is known to be 90 metres.



Calculate the approximate area of the block of land, using three applications of the Trapezoidal rule. *(3 marks)*

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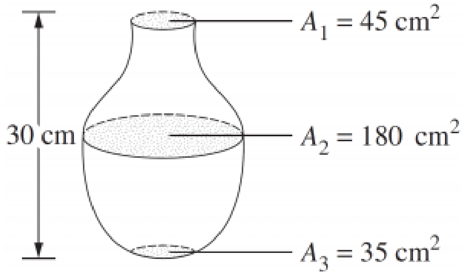
6. Measurement, STD2 M1 SM-Bank 17

Three equally spaced cross-sectional areas of a vase are shown.



Use the Trapezoidal rule to find the approximate capacity of the vase in litres. *(3 marks)*

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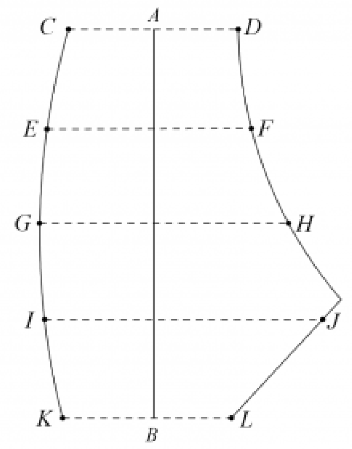
7. Measurement, STD2 M1 SM-Bank 19

An aerial diagram of a swimming pool is shown.



The swimming pool is a standard length of 50 metres but is not in the shape of a rectangle.

In the diagram of the swimming pool, the five widths are measured to be:

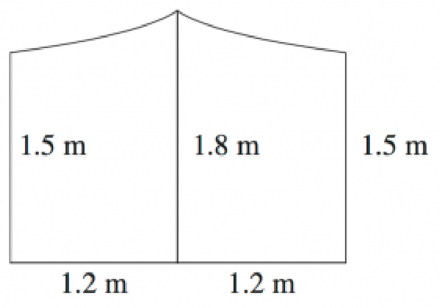
$$CD = 21.88\text{ m}$$
$$EF = 25.63\text{ m}$$
$$GH = 31.88\text{ m}$$
$$IJ = 36.25\text{ m}$$
$$KL = 21.88\text{ m}$$

(i) Use four applications of the Trapezoidal Rule to calculate the surface area of the pool. *(2 marks)*

(ii) The average depth of the pool is 1.2 m
Calculate the approximate volume of the swimming pool, in litres. *(1 mark)*

8. Measurement, STD2 M1 SM-Bank 01

The diagram shows the front of a tent supported by three vertical poles. The poles are 1.2 m apart. The height of each outer pole is 1.5 m, and the height of the middle pole is 1.8 m. The roof hangs between the poles.



The front of the tent has area A m².

(i) Use the trapezoidal rule to estimate A. *(2 marks)*

(ii) Explain whether the trapezoidal rule give a greater or smaller estimate of A? *(1 mark)*

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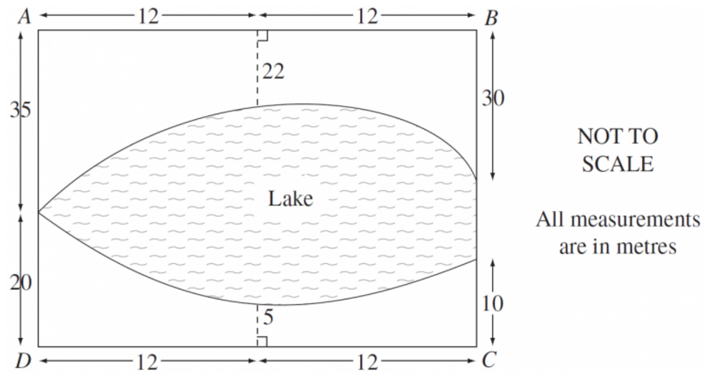
9. Measurement, STD2 M1 SM-Bank 18

There is a lake inside the rectangular grass picnic area $ABCD$, as shown in the diagram.



(i) Use Trapezoidal's Rule to find the approximate area of the lake's surface. *(3 marks)*

(ii) The lake is 60 cm deep. Bozo the clown thinks he can empty the lake using a four-litre bucket.

How many times would he have to fill his bucket from the lake in order to empty the lake? (Note that 1 m³ = 1000 L). *(2 marks)*

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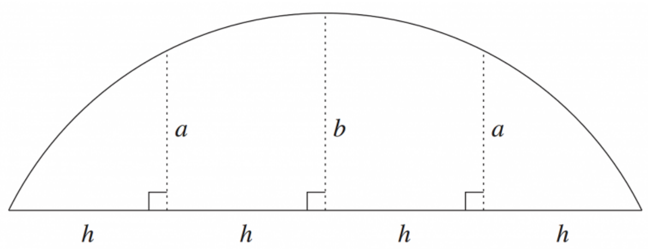
10. Measurement, STD2 M1 SM-Bank 20

A tunnel is excavated with a cross-section as shown.



(i) Find an expression for the area of the cross-section using the Trapezoidal rule. *(2 marks)*

(ii) The area of the cross-section must be 600 m^2. The tunnel is 80 m wide.

If the value of a increases by 2 metres, by how much will b change? *(2 marks)*

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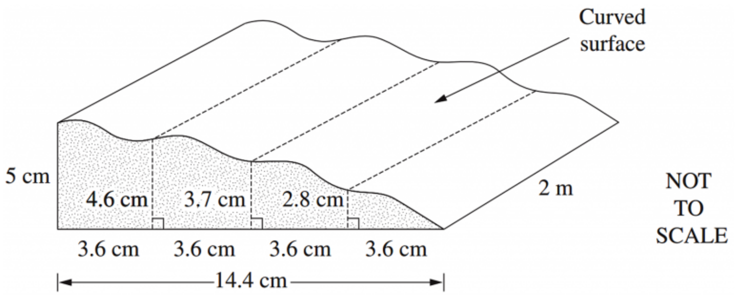
11. Measurement, STD2 M1 SM-Bank 22

A piece of plaster has a uniform cross-section, which has been shaded, and has dimensions as shown.



(i) Use the Trapezoidal rule to approximate the area of the cross-section. *(3 marks)*

(ii) The total surface area of the piece of plaster is 7480.8 cm^2

Calculate the area of the curved surface as shown on the diagram. Give your answer to the nearest square centimetre *(2 marks)*

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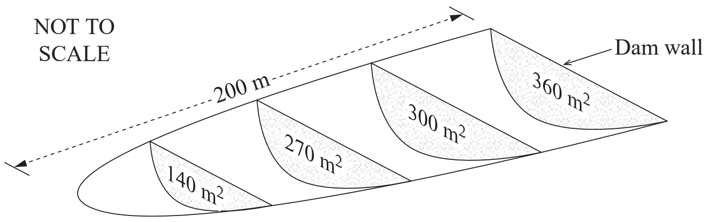
12. Measurement, STD2 M1 SM-Bank 21

A new 200-metre long dam is to be built.

The plan for the new dam shows evenly spaced cross-sectional areas.



(i) Using the Trapezoidal rule, show that the volume of the dam is approximately $44\,500$ m^3. *(2 marks)*

(ii) It is known that the catchment area for this dam is 2 km^2.

Assuming no wastage, calculate how much rainfall is needed, to the nearest mm, to fill the dam. *(2 marks)*

Worked Solutions

1. Measurement, STD2 M1 SM-Bank 15 MC

$$\text{Area} \approx \frac{20}{2}(23+15) + \frac{20}{2}(15+19)$$
$$\approx 10(38) + 10(34)$$
$$\approx 720 \text{ m}^2$$

$\Rightarrow$ A

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2. Measurement, STD2 M1 SM-Bank 16 MC

Solution 1

$$\text{Area} \approx \frac{3}{2}(6+7) + \frac{3}{2}(7+12) + \frac{3}{2}(12+8) + \frac{3}{2}(8+10)$$
$$\approx \frac{3}{2}(13+19+20+18)$$
$$\approx 105 \text{ m}^2$$

Solution 2

x	0	3	6	9	12
height	6	7	12	8	10
weight	1	2	2	2	1

$$\text{Area} \approx \frac{3}{2}(6 + 2\times 7 + 2\times 12 + 2\times 8 + 10)$$
$$\approx \frac{3}{2}(70)$$
$$\approx 105 \text{ m}^2$$

$\Rightarrow$ A

3. Measurement, STD2 2018 HSC 28a

Strategy 1

$$A \approx \frac{7.5}{2}(8.8+7.1)+\frac{7.5}{2}(7.1+9.8)+\frac{7.5}{2}(9.8+8.5)+\frac{7.5}{2}(8.5+4.9)$$
$$\approx 241.875$$
$$\approx 242 \text{ m}^2 \quad (\text{nearest m}^2)$$

Strategy 2

x	0	7.5	15	22.5	30
height	8.8	7.1	9.8	8.5	4.9
weight	1	2	2	2	1

$$A \approx \frac{7.5}{2}(8.8+2\times 7.1+2\times 9.8+2\times 8.5+4.9)$$
$$\approx 242 \text{ m}^2$$

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4. Measurement, STD2 M1 SM-Bank 02

Solution 1

Height $= 20 \div 4 = 5$ m

$$\text{Area} \approx \frac{5}{2}(28+18)+\frac{5}{2}(18+17)+\frac{5}{2}(17+16)+\frac{5}{2}(16+18)$$
$$\approx 370 \text{ m}^2$$

Solution 2

x	0	5	10	15	20
height	28	18	17	16	18
weight	1	2	2	2	1

$$\text{Area} \approx \frac{h}{2}[28+2(18+17+16)+18]$$
$$\approx \frac{5}{2}\times 148$$
$$\approx 370 \text{ m}^2$$

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5. Measurement, STD2 M1 SM-Bank 12

Solution 1

6 cm $\rightarrow$ 90 metres

1 cm $\rightarrow$ 15 metres

Height $= 2 \times 15 = 30$ metres

$$\text{Area} \approx \frac{30}{2}(75+60) + \frac{30}{2}(60+45) + \frac{30}{2}(45+60)$$
$$\approx 15(135+105+105)$$
$$\approx 5175 \text{ m}^2$$

Solution 2

After converting from scale:

x	0	30	60	90
height	75	60	45	60
weight	1	2	2	1

$$\text{Area} \approx \frac{30}{2}(75 + 2 \times 60 + 2 \times 45 + 60)$$
$$\approx 5175 \text{ m}^2$$

6. Measurement, STD2 M1 SM-Bank 17

Solution 1

$$V \approx \frac{15}{2}(45+180) + \frac{15}{2}(180+35)$$
$$\approx \frac{15}{2}(225+215)$$
$$\approx 3300 \text{ mL} \quad (1 \text{ cm}^3 = 1 \text{ mL})$$
$$\approx 3.3 \text{ L}$$

Solution 2

x	0	15	30
height	45	180	35
weight	1	2	1

$$V \approx \frac{15}{2}(45 + 2 \times 180 + 35)$$
$$\approx \frac{15}{2}(440)$$
$$\approx 3300 \text{ mL}$$
$$\approx 3.3 \text{ L}$$

7. Measurement, STD2 M1 SM-Bank 19

(i)

x	0	12.5	25	37.5	50
height	21.88	25.63	31.88	36.25	21.88
weight	1	2	2	2	1

Surface Area of pool

$$\approx \frac{12.5}{2}[21.88 + 2(25.63 + 31.88 + 36.25) + 21.88]$$

$$\approx 1445.5 \text{ m}^2$$

(ii) $V = Ah$

$$\approx 1445.5 \times 1.2$$

$$\approx 1734.6 \text{ m}^3$$

$$\approx 1\,734\,600 \text{ L}$$

♦ *Mean mark 50%. Be careful not to give away easy marks!*

8. Measurement, STD2 M1 SM-Bank 01

(i) $A \approx \dfrac{h}{2}[y_0 + 2y_1 + y_2]$

$$\approx \frac{1.2}{2}[1.5 + (2 \times 1.8) + 1.5]$$

$$\approx 0.6[6.6]$$

$$\approx 3.96 \text{ m}^2$$

(ii) The trapezoidal rule assumes a straight line between all points and therefore would estimate a greater area than the actual area of the tent front.

9. Measurement, STD2 M1 SM-Bank 18

(i) Area of lake = Area of rectangle − Area of grass

$$\text{Area of rectangle} = 24 \times 55$$

$$= 1320 \text{ m}^2$$

Area of grass (two applications)

$$\approx \frac{12}{2}(20 + 5) + \frac{12}{2}(5 + 10) + \frac{12}{2}(35 + 22) + \frac{12}{2}(22 + 30)$$

$$\approx 6(25 + 15 + 57 + 52)$$

$$\approx 894 \text{ m}^2$$

$$\therefore \text{ Area of lake} \approx 1320 - 894$$

$$\approx 426 \text{ m}^2$$

(ii) $V = Ah$

$$= 426 \times 0.6$$

$$= 255.6 \text{ m}^3$$

$$= 255\,600 \text{ L} \quad (1 \text{ m}^3 = 1000 \text{ L})$$

$$\therefore \text{ Times to fill bucket} = 255\,600 \div 4$$

$$= 63\,900$$

♦ *Mean mark 44%*

STRATEGY: Most students who did calculations in cm² and cm³ made errors. Keeping calculations in metres is ***much*** easier here.

10. Measurement, STD2 M1 SM-Bank 20

(i)

x	0	h	$2h$	$3h$	$4h$
height	0	a	b	a	0
weight	1	2	2	2	1

$$A \approx \frac{h}{2}[0 + 2(a + b + a) + 0]$$

$$\approx \frac{h}{2}(4a + 2b)$$

$$\approx h(2a + b)$$

(ii) $A = 600\text{ m}^2$

If tunnel is 80 metres wide

$4h = 80$

$h = 20$

Using part (i):

$600 = 20(2a + b)$

$2a + b = 30$

$b = 30 - 2a$

$\therefore$ If a increases by 2, b must decrease by 4.

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11. Measurement, STD2 M1 SM-Bank 22

(i)

x	0	3.6	7.2	10.8	14.4
height	5	4.6	3.7	2.8	0
weight	1	2	2	2	1

$$A \approx \frac{3.6}{2}[5 + 2(4.6 + 3.7 + 2.8) + 0]$$

$$\approx 1.8(27.2)$$

$$\approx 48.96\text{ cm}^2$$

(ii) Total Area $= 7480.8\text{ cm}^2$ (given)

Area of Base $= 14.4 \times 200$

$= 2880\text{ cm}^2$

Area of End $= 5 \times 200$

$= 1000\text{ cm}^2$

Area of sides $= 2 \times 48.96$

$= 97.92\text{ cm}^2$

$\therefore$ Area of curved surface

$= 7480.8 - (2880 + 1000 + 97.92)$

$= 3502.88$

$= 3503\text{ cm}^2$ (nearest cm^2)

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12. Measurement, STD2 M1 SM-Bank 21

(i)

x	0	50	100	150	200
height	360	300	270	140	0
weight	1	2	2	2	1

$$V \approx \frac{50}{2}[360 + 2(300 + 270 + 140) + 0]$$
$$\approx 25(1780)$$
$$\approx 44\ 500 \text{ m}^3$$

(ii) Convert $2 \text{ km}^2 \rightarrow \text{m}^2$:

$$2 \text{ km}^2 = 2000 \text{ m} \times 1000 \text{ m}$$
$$= 2\ 000\ 000 \text{ m}^2$$

♦♦♦ *Mean mark 11%.*

Using $V = Ah$ where $h = \text{rainfall}$:

$$44\ 500 = 2\ 000\ 000 \times h$$
$$\therefore h = \frac{44\ 500}{2\ 000\ 000}$$
$$= 0.02225\ldots \text{ m}$$
$$= 22.25\ldots \text{ mm}$$
$$= 22 \text{ mm} \text{ (nearest mm)}$$