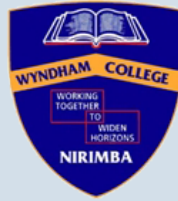


STD 1: Measurement (Std 1)
M3 Right-Angled Triangles (Y12)

Teacher: Kirtana Hariharan

Exam Equivalent Time: 66 minutes (based on HSC allocation of 1.5 minutes approx. per mark)



IMPORTANT FEATURES AND TIPS FROM 2UG EXAM HISTORY

- MS-M3 *Right-Angled Triangles* is a Year 12 Standard 1 topic.
- It has been a minor contributor to Gen2 past papers, although the absence of non-right angled trigonometry in the Standard 1 syllabus create the likelihood its contribution will increase meaningfully, in our view.

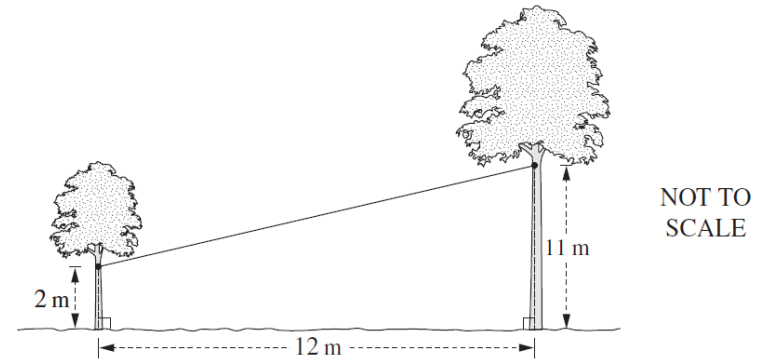
ANALYSIS - Common pitfalls

- *Right-Angled Triangles* is most commonly tested via multiple choice questions although longer answer style (i.e. multi-mark) questions have sporadically appeared, such as in 2017 and 2014.
- Students must be confident in applying all trig ratios, compass and true bearings and be well versed in common terminology such as *angle of depression* and *angle of elevation*.
- This sub-topic is a prime area to pick up marks, but note that questions in 2015 (Q9 MC), 2012 (Q27d) and 2009 (Q23a) produced sub-50% mean marks and deserve attention.
- Rounding an angle *to the nearest minute* is specifically mentioned in the new syllabus and warrants attention (when this was asked in 2017, it surprised many observers).
- In response to the above point, we have adjusted "nearest degree" in some past HSC questions to "nearest minute", testing students with some harder rounding examples.

Questions

1. Measurement, 2UG 2011 HSC 9 MC

Two trees on level ground, 12 metres apart, are joined by a cable. It is attached 2 metres above the ground to one tree and 11 metres above the ground to the other.

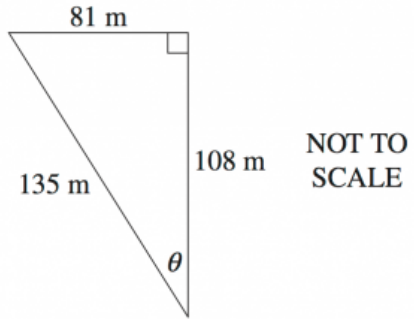


What is the length of the cable between the two trees, correct to the nearest metre?

- (A) 9 m
- (B) 12 m
- (C) 15 m
- (D) 16 m

2. Measurement, 2UG 2013 HSC 4 MC

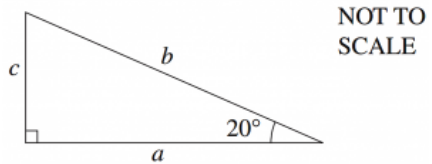
What is the value of θ , to the nearest degree?



- (A) 31°
 - (B) 37°
 - (C) 49°
 - (D) 53°
-

3. Measurement, 2UG 2004 HSC 5 MC

What is the correct expression for $\tan 20^\circ$ in this triangle?

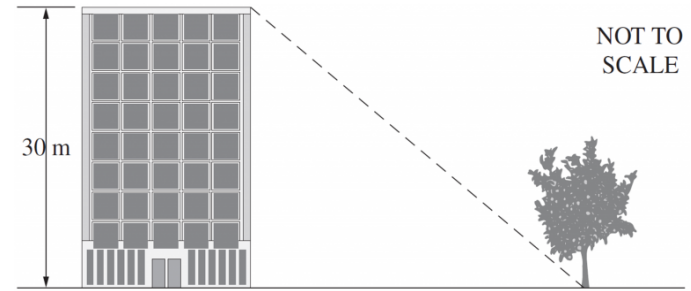


- (A) $\frac{a}{b}$
 - (B) $\frac{a}{c}$
 - (C) $\frac{c}{b}$
 - (D) $\frac{c}{a}$
-

4. Measurement, 2UG 2006 HSC 3 MC

The angle of depression of the base of the tree from the top of the building is 65° . The height of the building is 30 m.

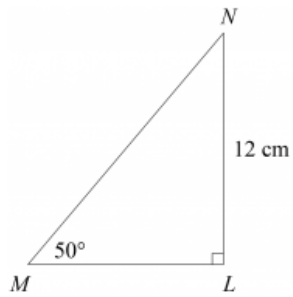
How far away is the base of the tree from the building, correct to one decimal place?



- (A) 12.7 m
 - (B) 14.0 m
 - (C) 33.1 m
 - (D) 64.3 m
-

5. Measurement, 2UG 2007 HSC 8 MC

What is the length of the side MN in the following triangle, correct to two decimal places?



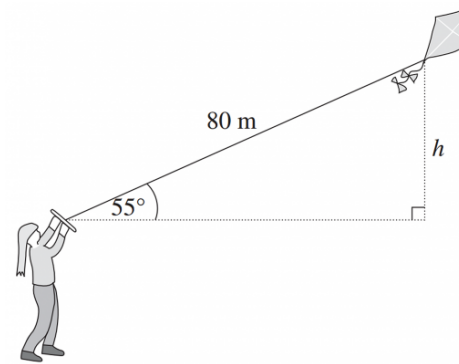
NOT
TO
SCALE

- (A) 9.19 cm
 - (B) 10.07 cm
 - (C) 15.66 cm
 - (D) 18.67 cm
-

6. Measurement, 2UG 2008 HSC 14 MC

Danni is flying a kite that is attached to a string of length 80 metres. The string makes an angle of 55° with the horizontal.

How high, to the nearest metre, is the kite above Danni's hand?



NOT
TO
SCALE

- (A) 46 m
 - (B) 66 m
 - (C) 98 m
 - (D) 114 m
-

7. Measurement, 2UG 2005 HSC 8 MC

If $\tan \theta = 85$, what is the value of θ , correct to 2 decimal places?

- (A) 1.37°
 - (B) 1.56°
 - (C) 89.33°
 - (D) 89.20°
-

8. Measurement, STD2 M6 SM-Bank 3 MC

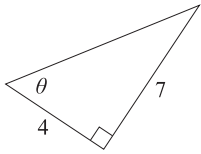
Which of the following expresses $S65^\circ W$ as a true bearing?

- A. 065°
 - B. 155°
 - C. 245°
 - D. 295°
-

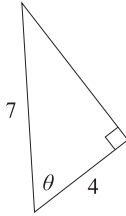
9. Measurement, STD2 M6 SM-Bank 01 MC

In which triangle is $\sin \theta = \frac{4}{7}$?

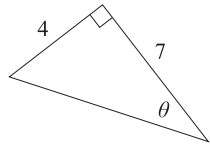
A.



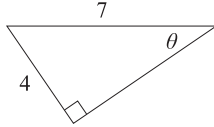
B.



C.



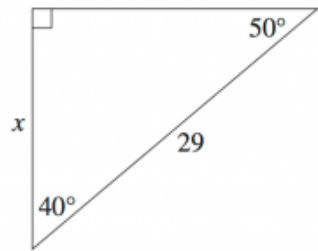
D.



10. Measurement, 2UG 2012 HSC 4 MC

⚡ RAP Data - Bottom 24%: School result (60%) was 7% above state average (53%)

Which expression could be used to calculate the value of x in this triangle?



NOT TO SCALE

(A) $29 \times \cos 40^\circ$

(B) $29 \times \cos 50^\circ$

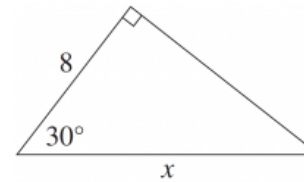
(C) $\frac{\cos 40^\circ}{29}$

(D) $\frac{\cos 50^\circ}{29}$

11. Measurement, 2UG 2009 HSC 4 MC

⚡ RAP Data - Bottom 14%: School result (65%) was 3% above state average (62%)

Which is the correct expression for the value of x in this triangle?



NOT TO SCALE

(A) $\frac{8}{\cos 30^\circ}$

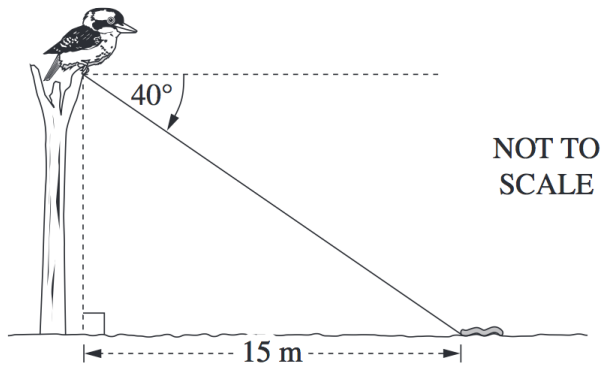
(B) $\frac{8}{\sin 30^\circ}$

(C) $8 \times \cos 30^\circ$

(D) $8 \times \sin 30^\circ$

12. Measurement, 2UG 2011 HSC 4 MC

The angle of depression from a kookaburra's feet to a worm on the ground is 40° . The worm is 15 metres from a point on the ground directly below the kookaburra's feet.

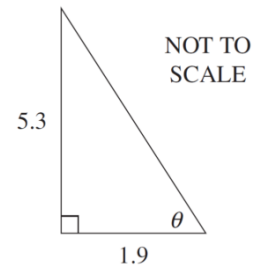


How high above the ground are the kookaburra's feet, correct to the nearest metre?

- (A) 10 m
 - (B) 11 m
 - (C) 13 m
 - (D) 18 m
-

13. Measurement, 2UG 2017 HSC 8 MC

The diagram shows a right-angled triangle.



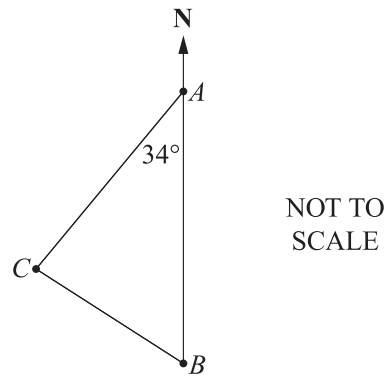
What is the value of θ , to the nearest minute?

- A. $70^\circ 16'$
 - B. $70^\circ 17'$
 - C. $70^\circ 27'$
 - D. $70^\circ 28'$
-

14. Measurement, 2UG 2018 HSC 7 MC

The diagram shows the positions of towns A , B and C .

Town A is due north of town B and $\angle CAB = 34^\circ$

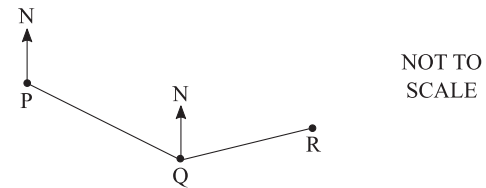


What is the bearing of town C from town A ?

- A. 034°
 - B. 146°
 - C. 214°
 - D. 326°
-

15. Measurement, STD2 M6 SM-Bank 4 MC

Ralph travels from P to Q on a bearing of 130° . He then turns and walks to R on a bearing of 075° .



What is the size of $\angle PQR$?

- A. 95°
 - B. 100°
 - C. 115°
 - D. 125°
-

16. Measurement, STD2 M6 SM-Bank 7 MC

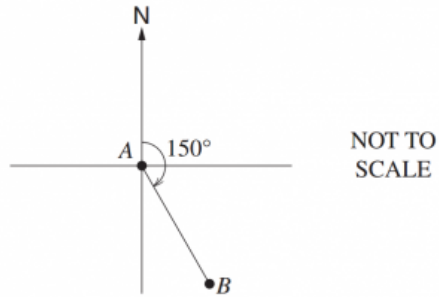
Jeet walks 5 km from his home on a bearing of 153° . He then walks due north until he arrives a point which is due east of his home.

How far east, to the nearest 0.1 km, is Jeet from home?

- A. 2.3 km
 - B. 2.5 km
 - C. 4.9 km
 - D. 9.8 km
-

17. Measurement, 2UG 2010 HSC 10 MC

A plane flies on a bearing of 150° from A to B .

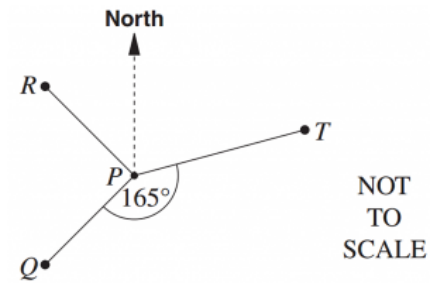


What is the bearing of A from B ?

- (A) 30°
 - (B) 150°
 - (C) 210°
 - (D) 330°
-

18. Measurement, 2UG 2008 HSC 17 MC

The diagram shows the position of Q , R and T relative to P .



In the diagram,

- Q is south-west of P
- R is north-west of P
- $\angle QPT$ is 165°

What is the bearing of T from P ?

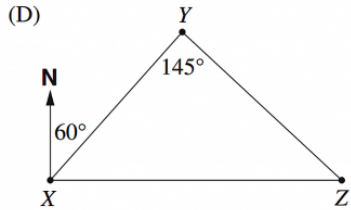
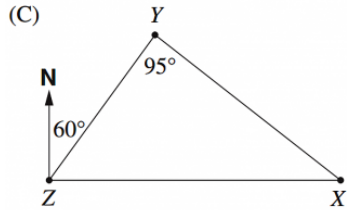
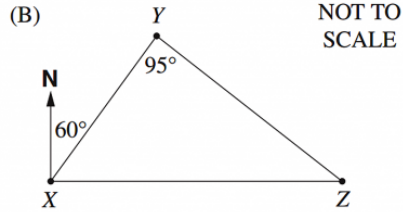
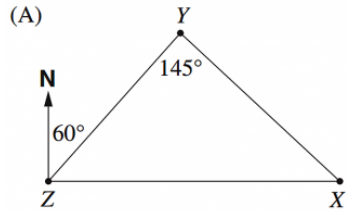
- (A) 060°
 - (B) 075°
 - (C) 105°
 - (D) 120°
-

19. Measurement, 2UG 2014 HSC 23 MC

The following information is given about the locations of three towns X , Y and Z :

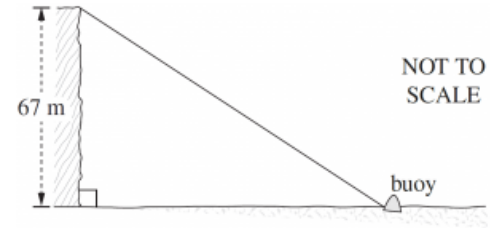
- X is due east of Z
- X is on a bearing of 145° from Y
- Y is on a bearing of 060° from Z .

Which diagram best represents this information?



20. Measurement, 2UG 2015 HSC 9 MC

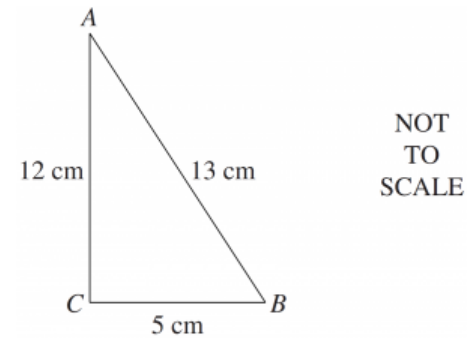
From the top of a cliff 67 metres above sea level, the angle of depression of a buoy is 42° .



How far is the buoy from the base of the cliff, to the nearest metre?

- (A) 60 m
 (B) 74 m
 (C) 90 m
 (D) 100 m

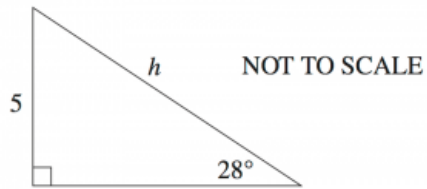
21. Measurement, 2UG 2005 HSC 25b



- (i) Use Pythagoras' theorem to show that $\triangle ABC$ is a right-angled triangle. (1 mark)
 (ii) Calculate the size of $\angle ABC$ to the nearest **minute**. (2 marks)

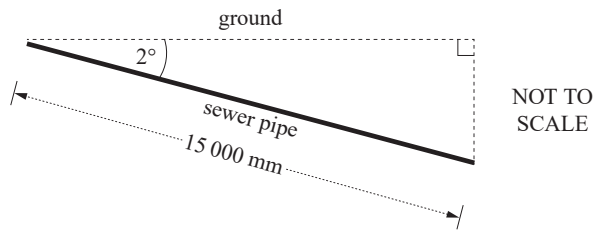
22. Measurement, 2UG 2014 HSC 26b

Calculate the value of h correct to two decimal places. (2 marks)



23. Measurement, 2UG 2017 HSC 26d

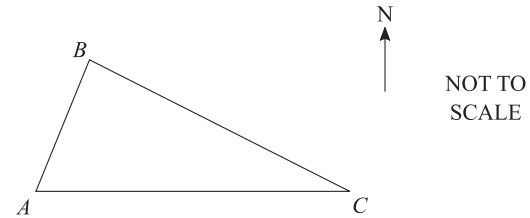
A sewer pipe needs to be placed into the ground so that it has a 2° angle of depression. The length of the pipe is 15 000 mm.



How much deeper should one end of the pipe be compared to the other end? Answer to the nearest mm. (2 marks)

24. Measurement, STD2 M6 SM-Bank 4

The diagram shows three checkpoints A , B and C . Checkpoint C is due east of Checkpoint A . The bearing of Checkpoint B from Checkpoint A is $N22^\circ E$ and the bearing of Checkpoint C from Checkpoint B is $S68^\circ E$. The distance between Checkpoint A and Checkpoint B is 42 kilometres.

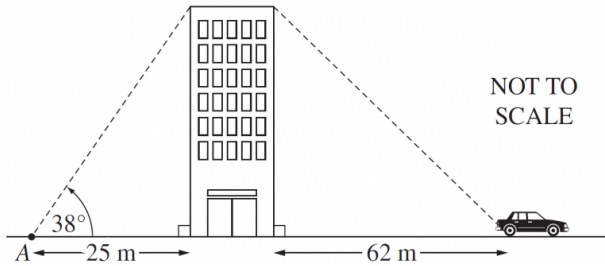


- Mark the given information on the diagram and explain why $\angle ABC$ is 90° . (2 marks)
- Find the distance, to the nearest kilometre, between Checkpoint A and Checkpoint B . (2 marks)
- If a runner is travelling 12.6 km/h, how long does it take her to travel between Checkpoint A and Checkpoint B , in hours and minutes? (2 marks)

25. Measurement, 2UG 2009 HSC 23a

⚡ Part ii: RAP Data - Bottom 22%: School result (39%) was 6% above state average (33%)

The point A is 25 m from the base of a building. The angle of elevation from A to the top of the building is 38° .

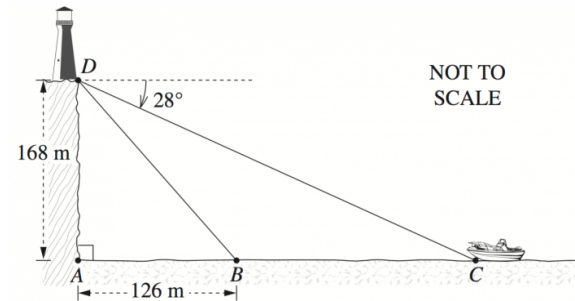


- Show that the height of the building is approximately 19.5 m. (1 mark)
- A car is parked 62 m from the base of the building.

What is the angle of depression from the top of the building to the car?
Give your answer to the nearest **minute**. (2 marks)

26. Measurement, 2UG 2010 HSC 24d

The base of a lighthouse, D , is at the top of a cliff 168 metres above sea level. The angle of depression from D to a boat at C is 28° . The boat heads towards the base of the cliff, A , and stops at B . The distance AB is 126 metres.

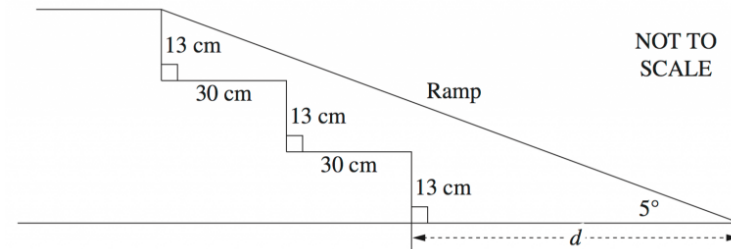


- What is the angle of depression from D to B , correct to the nearest degree? (3 marks)
- How far did the boat travel from C to B , correct to the nearest metre? (2 marks)

27. Measurement, 2UG 2012 HSC 27d

A disability ramp is to be constructed to replace steps, as shown in the diagram.

The angle of inclination for the ramp is to be 5° .



Calculate the extra distance, d , that the ramp will extend beyond the bottom step.

Give your answer to the nearest centimetre. (3 marks)

Worked Solutions

1. Measurement, 2UG 2011 HSC 9 MC

Using Pythagoras

$$\begin{aligned}c^2 &= 12^2 + 9^2 \\ &= 144 + 81 \\ &= 225\end{aligned}$$

$$\therefore c = 15, \quad c > 0$$

$\Rightarrow C$

2. Measurement, 2UG 2013 HSC 4 MC

$$\sin \theta = \frac{81}{135}$$

$$\therefore \theta = 37^\circ$$

$\Rightarrow B$

Note that $\tan \theta$ or $\cos \theta$ could also be used.

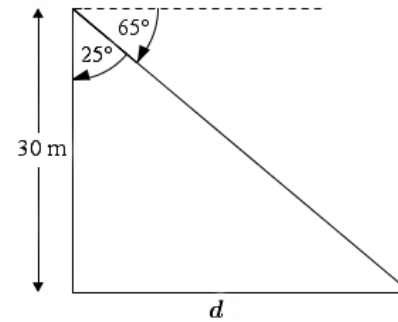
3. Measurement, 2UG 2004 HSC 5 MC

$$\begin{aligned}\tan 20^\circ &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{c}{a}\end{aligned}$$

$\Rightarrow D$

Worked Solutions

4. Measurement, 2UG 2006 HSC 3 MC



Let d = distance from base to tree

$$\tan 25^\circ = \frac{d}{30}$$

$$\begin{aligned}\therefore d &= 30 \times \tan 25^\circ \\ &= 13.98 \dots \text{ m}\end{aligned}$$

$\Rightarrow B$

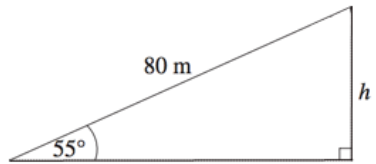
5. Measurement, 2UG 2007 HSC 8 MC

$$\sin 50^\circ = \frac{12}{MN}$$

$$\begin{aligned}MN &= \frac{12}{\sin 50^\circ} \\ &= 15.66 \dots\end{aligned}$$

$\Rightarrow C$

6. Measurement, 2UG 2008 HSC 14 MC



$$\sin 55^\circ = \frac{h}{80}$$

$$h = 80 \times \sin 55^\circ$$

$$= 65.532\dots\text{ m}$$

$\Rightarrow B$

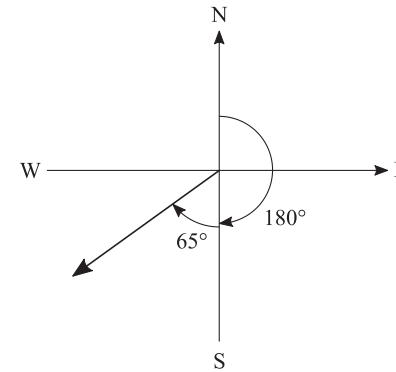
7. Measurement, 2UG 2005 HSC 8 MC

$$\tan \theta = 85$$

$$\theta = \tan^{-1} 85$$

$$= 89.33^\circ$$

8. Measurement, STD2 M6 SM-Bank 3 MC

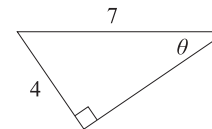


$$\text{True bearing} = 180 + 65$$

$$= 245^\circ$$

$\Rightarrow C$

9. Measurement, STD2 M6 SM-Bank 01 MC



$\Rightarrow D$

10. Measurement, 2UG 2012 HSC 4 MC

$$\cos 40^\circ = \frac{x}{29}$$

$$x = 29 \times \cos 40^\circ$$

$\Rightarrow A$

♦ Mean mark 42%

11. Measurement, 2UG 2009 HSC 4 MC

$$\cos 30^\circ = \frac{8}{x}$$

$$\therefore x = \frac{8}{\cos 30^\circ}$$

$\Rightarrow A$

12. Measurement, 2UG 2011 HSC 4 MC

\angle Elevation (worm) = 40° (alternate angles)

$$\tan 40^\circ = \frac{h}{15}$$

$$h = 15 \times \tan 40^\circ$$

$$= 12.58... \text{ m}$$

$\Rightarrow C$

13. Measurement, 2UG 2017 HSC 8 MC

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$= \frac{5.3}{1.9}$$

$$= 2.789...$$

$$\therefore \theta = 70.277...^\circ$$

$$= 70^\circ 16' 39.8''$$

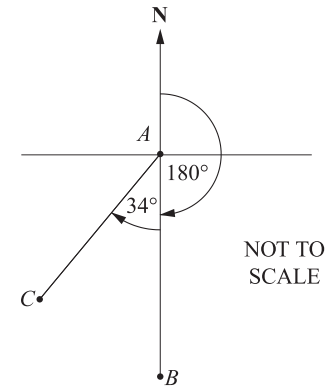
$$= 70^\circ 17'$$

$\Rightarrow B$

COMMENT: An angle that has over $30''$ (seconds) is rounded up to the next minute (i.e. rounded up to $70^\circ 17'$).

14. Measurement, 2UG 2018 HSC 7 MC

Bearing of Town C from Town A :

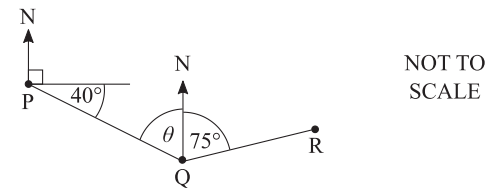


$$\text{Bearing} = 180 + 34$$

$$= 214^\circ$$

$\Rightarrow C$

15. Measurement, STD2 M6 SM-Bank 4 MC



$$\theta = 90 - 40 \quad (180^\circ \text{ in } \Delta)$$

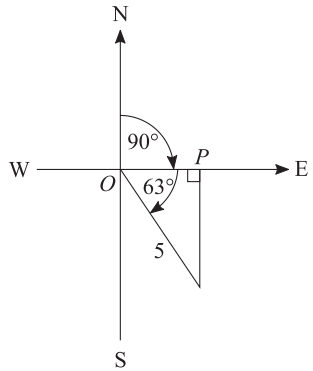
$$= 50^\circ$$

$$\therefore \angle PQR = 50 + 75$$

$$= 125^\circ$$

$\Rightarrow D$

16. Measurement, STD2 M6 SM-Bank 7 MC



Jeet finishes at P

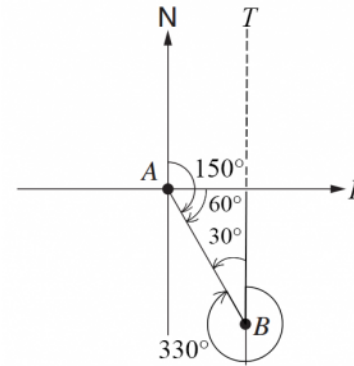
Find OP :

$$\cos 63^\circ = \frac{OP}{5}$$

$$\begin{aligned} \therefore OP &= 5 \times \cos 63^\circ \\ &= 2.26\dots \end{aligned}$$

$\Rightarrow A$

17. Measurement, 2UG 2010 HSC 10 MC



♦♦ Mean mark 34%

$$\angle TBA = 30^\circ \text{ (angle sum of triangle)}$$

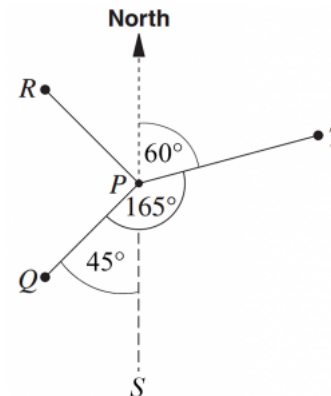
$$\therefore \text{Bearing of } A \text{ from } B$$

$$= 360 - 30$$

$$= 330^\circ$$

$\Rightarrow D$

18. Measurement, 2UG 2008 HSC 17 MC



$$\angle QPS = 45^\circ \text{ (Q is south west of P)}$$

$$\angle TPS = 165 - 45 = 120^\circ$$

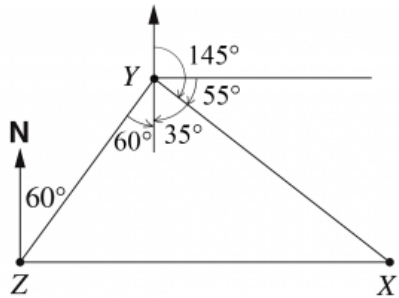
$$\therefore \angle NPT = 60^\circ \text{ (180^\circ in straight line)}$$

$\Rightarrow A$

19. Measurement, 2UG 2014 HSC 23 MC

Since X is due east of Z

\Rightarrow Cannot be B or D



The diagram shows we can find

$$\angle ZYX = 60 + 35 = 95^\circ$$

Using alternate angles (60°) and

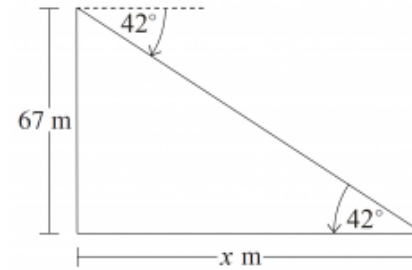
the 145° bearing of X from Y

$\Rightarrow C$

♦ Mean mark 38%

COMMENT: Drawing a parallel North/South line through Y makes this question *much simpler* to solve.

20. Measurement, 2UG 2015 HSC 9 MC



♦ Mean mark 49%

COMMENT: The angle of depression is a regularly examined concept. Make sure you know exactly what it refers to.

Let x = distance of buoy from cliff base

$$\tan 42^\circ = \frac{67}{x}$$

$$x \tan 42^\circ = 67$$

$$x = \frac{67}{\tan 42^\circ}$$

$$= 74.41 \dots \text{ m}$$

$\Rightarrow B$

21. Measurement, 2UG 2005 HSC 25b

(i) $\triangle ABC$ is right-angled if $a^2 + b^2 = c^2$

$$a^2 + b^2 = 5^2 + 12^2$$

$$= 169$$

$$= 13^2$$

$$= c^2 \dots \text{ as required.}$$

(ii) $\sin \angle ABC = \frac{12}{13}$

$$\therefore \angle ABC = 67.38 \dots^\circ$$

$$= 67^\circ 22' 48''$$

$$= 67^\circ 23' \text{ (nearest minute)}$$

MARKER'S COMMENT: Know your calculator process for producing an angle in minutes/seconds. Note >30 "seconds" rounds up to the higher "minute"!

22. Measurement, 2UG 2014 HSC 26b

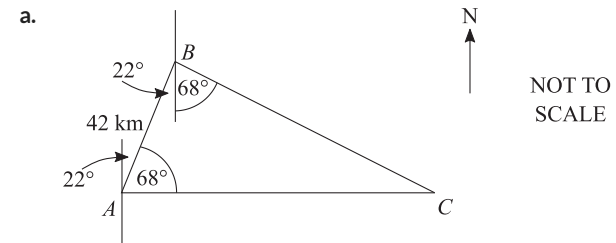
$$\begin{aligned}\sin 28^\circ &= \frac{5}{h} \\ \therefore h &= \frac{5}{\sin 28^\circ} \\ &= 10.6502\dots \\ &= 10.65 \text{ (2 d.p.)}\end{aligned}$$

23. Measurement, 2UG 2017 HSC 26d

Let x = depth needed

$$\begin{aligned}\sin 2^\circ &= \frac{x}{15\,000} \\ x &= 15\,000 \times \sin 2^\circ \\ &= 523.49\dots \\ &= 523 \text{ mm (nearest mm)}\end{aligned}$$

24. Measurement, STD2 M6 SM-Bank 4



$$\begin{aligned}\angle ABC &= 22 + 68 \\ &= 90^\circ\end{aligned}$$

b. In $\triangle ABC$,

$$\begin{aligned}\cos \angle BAC &= \frac{AB}{AC} \\ \cos 68^\circ &= \frac{42}{AC} \\ AC &= \frac{42}{\cos 68^\circ} \\ &= 112.11\dots \\ &= 112 \text{ km (nearest km)}\end{aligned}$$

c. Travel time = $\frac{\text{dist}}{\text{speed}}$

$$\begin{aligned}&= \frac{42}{12.6} \\ &= 3.333\dots \\ &= 3\text{h } 20 \text{ mins}\end{aligned}$$

25. Measurement, 2UG 2009 HSC 23a

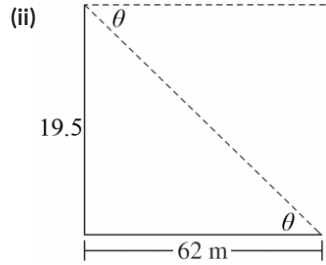
(i) Need to prove height (h) \approx 19.5 m

$$\tan 38^\circ = \frac{h}{25}$$

$$h = 25 \times \tan 38^\circ$$

$$= 19.5321\dots$$

$$\approx 19.5 \text{ m ... as required.}$$



Let \angle Elevation (from car) = θ

$$\tan \theta = \frac{h}{62}$$

$$= \frac{19.5}{62}$$

$$= 0.3145\dots$$

$$\theta = 17.459\dots^\circ$$

$$= 17^\circ 27' 33'' \dots$$

$$= 17^\circ 28' \text{ (nearest minute)}$$

♦♦ Mean mark 33%
MARKER'S COMMENT: If >30
 "seconds", round to the higher
 "minute".

$\therefore \angle$ Depression to car = $17^\circ 28'$ (alternate to θ)

26. Measurement, 2UG 2010 HSC 24d

(i) $\tan \angle ADB = \frac{126}{168}$

$$\angle ADB = 36.8698\dots$$

$$= 36.9^\circ \text{ (to 1 d.p)}$$

♦♦ Mean mark 31%

$$\angle \text{Depression } D \text{ to } B = 90 - 36.9$$

$$= 53.1$$

$$= 53^\circ \text{ (nearest degree)}$$

(ii) Find CB :

$$\angle ADC + 28 = 90$$

$$\angle ADC = 62^\circ$$

$$\tan 62^\circ = \frac{AC}{168}$$

$$AC = 168 \times \tan 62^\circ$$

$$= 315.962\dots$$

♦♦ Mean mark 31%
MARKER'S COMMENT: Solve efficiently by
 using right-angled trigonometry.
 Many students used non-right
 angled trig, adding to the
 calculations and the difficulty.

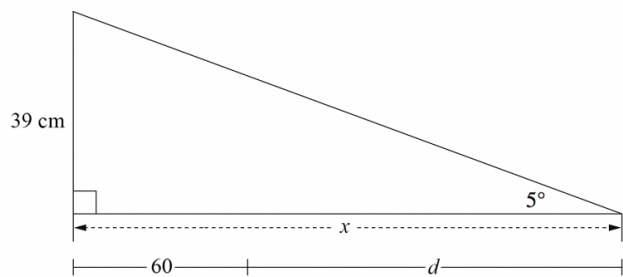
$$CB = AC - AB$$

$$= 315.962\dots - 126$$

$$= 189.962\dots$$

$$= 190 \text{ m (nearest m)}$$

27. Measurement, 2UG 2012 HSC 27d



Let the horizontal part of the ramp = x cm

$$\tan 5^\circ = \frac{39}{x}$$

$$\begin{aligned}x &= \frac{39}{\tan 5^\circ} \\ &= 445.772\dots\end{aligned}$$

♦♦ Mean mark 35%

MARKER'S COMMENT: The better responses used a diagram of a simplified version of the ramp as per the Worked Solution.

Since $x = 60 + d$

$$\begin{aligned}d &= 445.772 - 60 \\ &= 385.772 \text{ cm} \\ &= 386 \text{ cm (nearest cm)}\end{aligned}$$