STD 1: Networks (Std 1), N1 Networks and Paths (Y12)

Basic Concepts

Teacher: Kirtana Hariharan

Exam Equivalent Time: 45 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

OVERVIEW

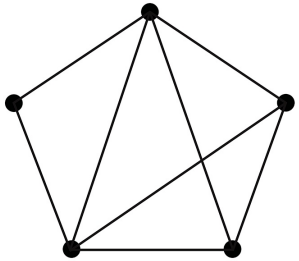
- *MS-N1 Networks and Paths - Basic Concepts* is a branch of mathematics never tested in the HSC's history, creating a significant and exciting challenge for teachers and students.
- Question concepts and terminology have been adjusted to reflect the latest information officially released and the advice of respected practitioners. This is an ongoing process.

ANALYSIS -

- **Network diagram, table, matrix:** Completing a table (or matrix) from a network graph and vice versa is a core competency we view as even *more* important after NESA's recent release of supplementary questions - this is reflected in our database. Question styles in this critical area are informed by official topic guidance and sample exam questions.
- **Basic Concepts:** Questions review basic concepts such as; odd and even vertices, tree and path definitions, connected network, weighted edge, etc..
- Multiple choice questions are used in our database to allow teachers to target specific basic concepts and evaluate student understanding (or lack thereof).
- **Practical problems:** a number of practical problems requiring simple network design and representation are included.
- The Konigsberg Bridge problem explicitly mentioned in the Standard 2 syllabus has been conspicuously removed from the similar section of the Standard 1 syllabus - this reduced content coverage has been reflected in the database.

Questions

1. Networks, FUR1 2010 VCE 2 MC



The number of edges in the graph above is

A. 5

B. 7

C. 8

D. 10

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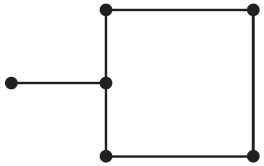
2. Networks, FUR1 2012 VCE 1 MC



The sum of the degrees of all the vertices in the graph above is

A. 6

B. 9

C. 11

D. 12

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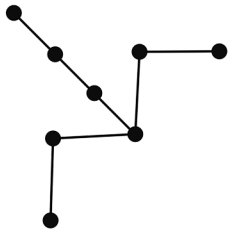
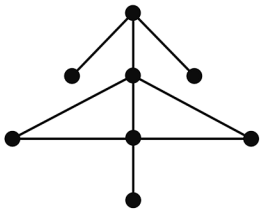
3. Networks, FUR1 2013 VCE 1 MC

Which one of the following graphs is a tree?

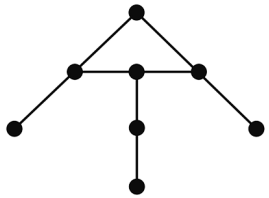
A.



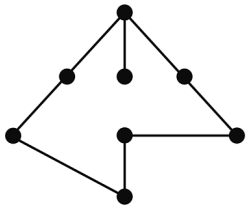
B.



C.



D.



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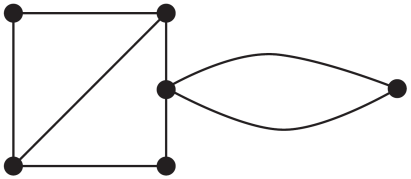
4. Networks, FUR1 2015 VCE 1 MC



In the graph above, the number of vertices of odd degree is

A. 1

B. 2

C. 3

D. 4

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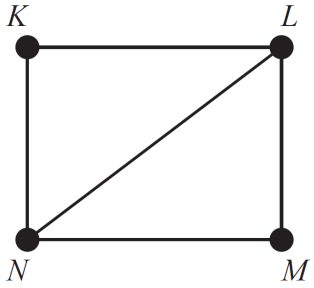
5. Networks, FUR1 2015 VCE 5 MC

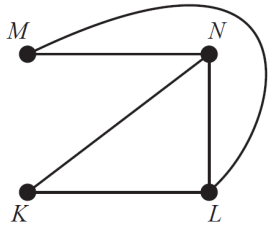
The graph below represents a friendship network. The vertices represent the four people in the friendship network: Kwan (*K*), Louise (*L*), Milly (*M*) and Narelle (*N*).

An edge represents the presence of a friendship between a pair of these people. For example, the edge connecting *K* and *L* shows that Kwan and Louise are friends.

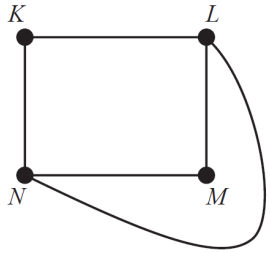


Which one of the following graphs does not contain the same information?

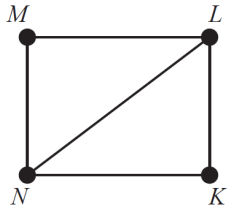
A.



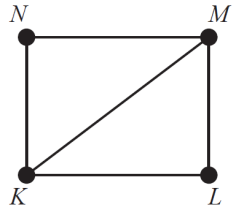
B.



C.



D.



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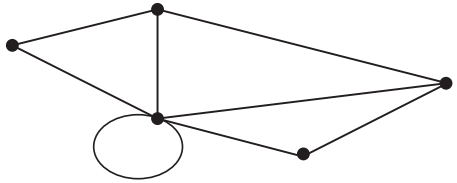
6. Networks, STD N2 SM-Bank 32 MC



The number of vertices with an odd degree in the network above is

A. 2

B. 3

C. 4

D. 5

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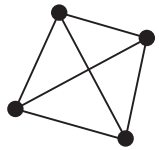
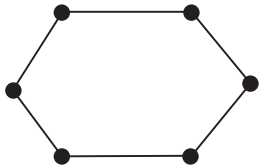
7. Networks, FUR1 2017 VCE 2 MC

Two graphs, labelled Graph 1 and Graph 2, are shown below.



Graph 1



Graph 2

The sum of the degrees of the vertices of Graph 1 is

A. two less than the sum of the degrees of the vertices of Graph 2.

B. one less than the sum of the degrees of the vertices of Graph 2.

C. equal to the sum of the degrees of the vertices of Graph 2.

D. two more than the sum of the degrees of the vertices of Graph 2.

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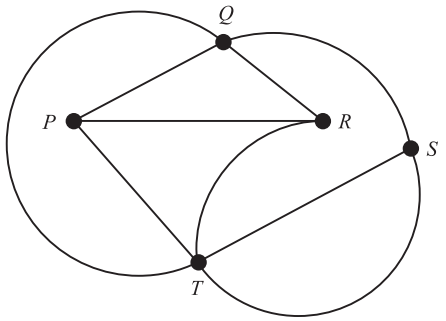
8. Networks, FUR1 2018 VCE 04 MC

Consider the graph below.



Which one of the following is **not** a path for this graph?

A. $PRQTS$

B. $PRTSQ$

C. $PTQSR$

D. $PTRQS$

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9. Networks, STD2 N2 SM-Bank 03 MC

A store manager is directly in charge of five department managers.

Each department manager is directly in charge of six sales people in their department.

This staffing structure could be represented graphically by

A. a tree.

B. a path.

C. a cycle.

D. a weighted graph.

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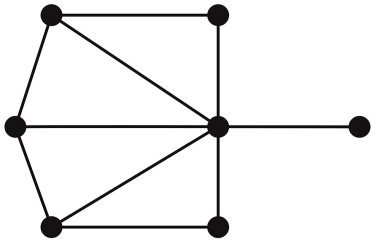
10. Networks, FUR1 2011 VCE 1 MC



In the network shown, the number of vertices of even degree is

A. 2

B. 3

C. 4

D. 5

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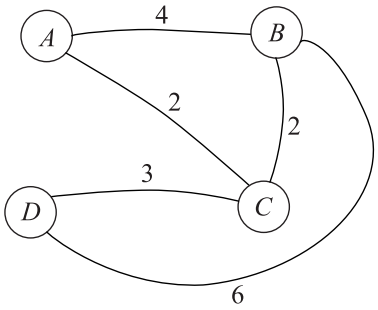
11. Networks, STD2 N2 SM-Bank 20

A table is constructed to represent the network diagram below.



Complete the table. ***(2 marks)***

	A	*B*	*C*	*D*
A	0	4		-
B	4	0		
C			0	3
D		6		0

---

12. Networks, STD2 N2 SM-Bank 28

In central Queensland, there are four petrol stations A, B, C and D. The table shows the length, in kilometres, of roads connecting these petrol stations.

	A	*B*	*C*	*D*
A	–	170	–	150
B	170	–	160	90
C	–	160	–	120
D	150	90	120	–

a. Construct a network diagram to represent the information in the table. ***(2 marks)***

b. A petrol tanker needs to refill each station. It starts at Station A and visits each station.
Calculate the shortest distance that can be travelled by the petrol tanker. In your answer, include the order the petrol stations are refilled. ***(2 marks)***

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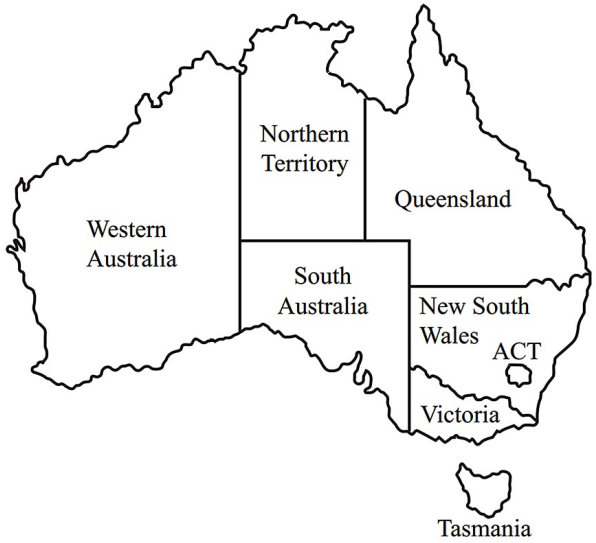
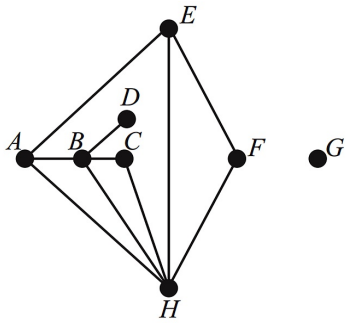
13. Networks, STD2 N2 SM-Bank 37

The map of Australia shows the six states, the Northern Territory and the Australian Capital Territory (ACT).



In the network diagram below, each of the vertices A to H represents one of the states or territories shown on the map of Australia. The edges represent a border shared between two states or between a state and a territory.



(i) In the network diagram, what is the order of the vertex that represents the Australian Capital Territory (ACT)? ***(1 mark)***

(ii) In the network diagram, Queensland is represented by which letter? Explain why. ***(2 marks)***

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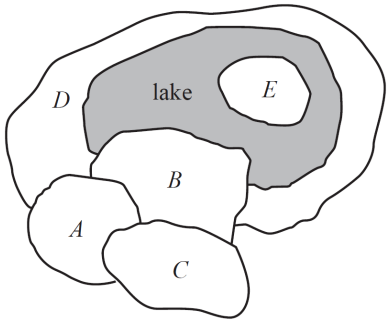
14. Networks, FUR2 2009 VCE 1

The city of Robville is divided into five suburbs labelled as A to E on the map below.

A lake which is situated in the city is shaded on the map.



An matrix is constructed to represent the number of land borders between suburbs.

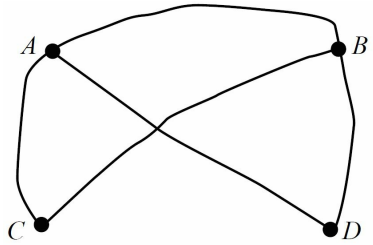
If there is no land border between two suburbs, the matrix records a '0'. If there is a single land border between two suburbs, the matrix records a '1', and if there are two separate land borders between the same two suburbs, the matrix records a '2'.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

a. Explain why all values in the final row and final column are zero. ***(1 mark)***

In the network diagram below, vertices represent suburbs and edges represent land borders between suburbs.

The diagram has been started but is not finished.



b. The network diagram is missing one edge and one vertex.

On the diagram

i. draw the missing edge *(1 mark)*

ii. draw and label the missing vertex. *(1 mark)*

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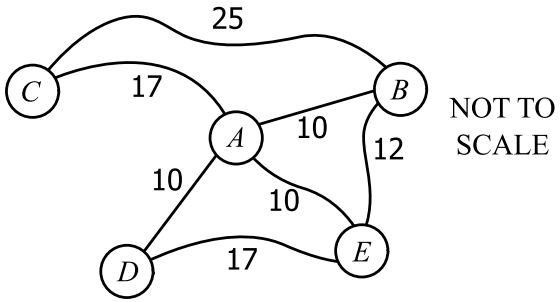
15. Networks, STD2 N2 SM-Bank 11

A network of roads between towns shows the travelling times in minutes between towns that are directly connected.



Complete the shaded cells in the following table so that it represents the information in this network. *(2 marks)*

	A	*B*	*C*	*D*	*E*
A	0	10		10	10
B		0			
C	17		0	—	
D		—	—	0	17
E	10				0

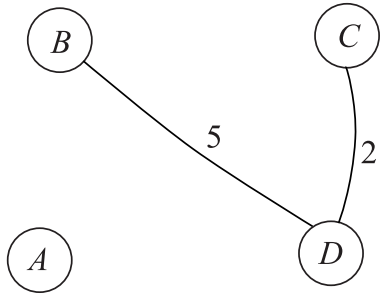
---

16. Networks, STD2 N2 SM-Bank 21

The table below represents a network.

	A	*B*	*C*	*D*
A	-	0	7	4
B	0	-	3	5
C	7	3	-	2
D	4	5	2	-

Complete the network diagram below to accurately reflect the information in the above table. *(2 marks)*



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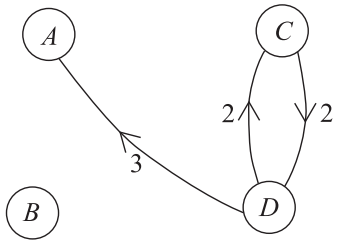
17. Networks, STD2 N2 SM-Bank 22

The table below represents a directed network.

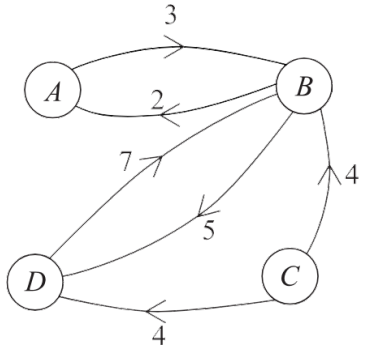
		To			
		A	*B*	*C*	*D*
From	*A*	0	4	-	4
	B	-	0	-	3
	C	-	6	0	2
	D	3	5	2	0

Complete the network diagram below to accurately reflect the network described in the above table. ***(2 marks)***



18. Networks, STD2 N2 SM-Bank 23

A directed network diagram is pictured below.



The information in the network diagram is used to complete the network table below, with a "0" used to signify that no connection exists. Complete the table. ***(2 marks)***

		To			
		A	*B*	*C*	*D*
From	*A*	-	3		0
	B	2	-		
	C			-	4
	D				-

Worked Solutions

1. Networks, FUR1 2010 VCE 2 MC

$\Rightarrow C$

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2. Networks, FUR1 2012 VCE 1 MC

Total Degrees

$= 1 + 3 + 2 + 2 + 2 + 2$

$= 12$

$\Rightarrow D$

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3. Networks, FUR1 2013 VCE 1 MC

A tree cannot contain a cycle.

$\Rightarrow A$

---

4. Networks, FUR1 2015 VCE 1 MC

$\Rightarrow B$

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5. Networks, FUR1 2015 VCE 5 MC

Option D has Kwan and Milly as friends which is not correct.

$\Rightarrow D$

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6. Networks, STD N2 SM-Bank 32 MC

$\Rightarrow A$

(Note a loop creates 2 extra degrees to a vertex.)

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7. Networks, FUR1 2017 VCE 2 MC

Graph 1

$\sum$ degrees $= 3 + 3 + 3 + 3 = 12$

Graph 2

$\sum$ degrees $= 2 + 2 + 2 + 2 + 2 + 2 = 12$

$\Rightarrow C$

---

8. Networks, FUR1 2018 VCE 04 MC

By trial and error:

Consider option C,

$PTQSR$ is not a path because S to R

must go through another vertex.

$\Rightarrow C$

---

9. Networks, STD2 N2 SM-Bank 03 MC

$\Rightarrow A$

---

10. Networks, FUR1 2011 VCE 1 MC

$\Rightarrow B$

---

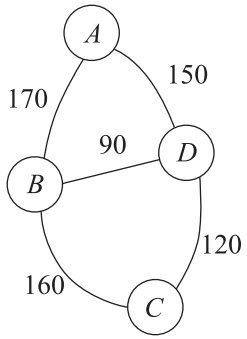
11. Networks, STD2 N2 SM-Bank 20

	A	*B*	*C*	*D*
A	0	4	**2**	-
B	4	0	**2**	**6**
C	**2**	**2**	0	3
D	-	6	**3**	0

Worked Solutions

12. Networks, STD2 N2 SM-Bank 28

a.



b. Shortest Path from A (visiting all stations)

$A - B - D - C$

$\text{Distance} = 170 + 90 + 120$

$= 380 \text{ km}$

13. Networks, STD2 N2 SM-Bank 37

(i) ACT has 1 border (with NSW)

$\therefore$ **Degree of ACT's vertex $= 1$**

(ii) NSW is Vertex B (it is connected to the ACT - Vertex D)

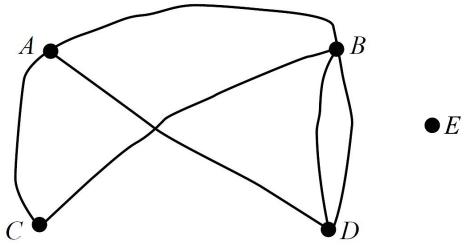
$\therefore$ **Queensland is vertex A as it is connected to B and has degree 3.**

(C is Victoria as it has degree 2)

14. Networks, FUR2 2009 VCE 1

a. E has no land borders with other suburbs.

b.i. & ii.

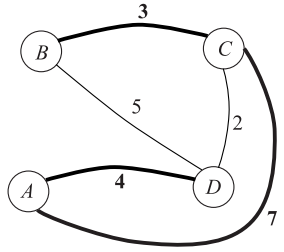


15. Networks, STD2 N2 SM-Bank 11

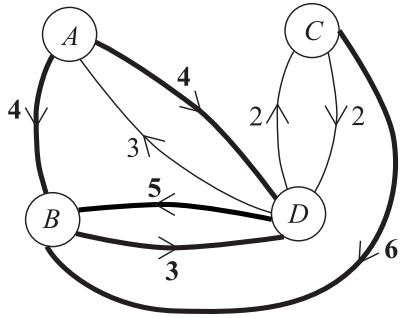
	A	B	C	D	E
A	0	10	**17**	10	10
B	**10**	0	**25**	—	**12**
C	17	**25**	0	—	—
D	**10**	—	—	0	17
E	10	**12**	—	**17**	0

Note the symmetry in this table across the diagonal.

16. Networks, STD2 N2 SM-Bank 21



17. Networks, STD2 N2 SM-Bank 22



18. Networks, STD2 N2 SM-Bank 23

		To			
		A	*B*	*C*	*D*
From	*A*	-	3	**0**	0
	B	2	-	**0**	**5**
	C	**0**	**4**	-	4
	D	**0**	**7**	**0**	-