STD 1: Networks (Std 1), N1 Networks and Paths (Y12)

Shortest Paths

Teacher: Kirtana Hariharan

Exam Equivalent Time: 37.5 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

OVERVIEW

- *MS-N1 Shortest Paths* is a sub-topic of *Networks.*
- This new content area has no established HSC exam history, and as such, we have created a database from a wide range of high quality sources, including all official NESA releases on the topic.

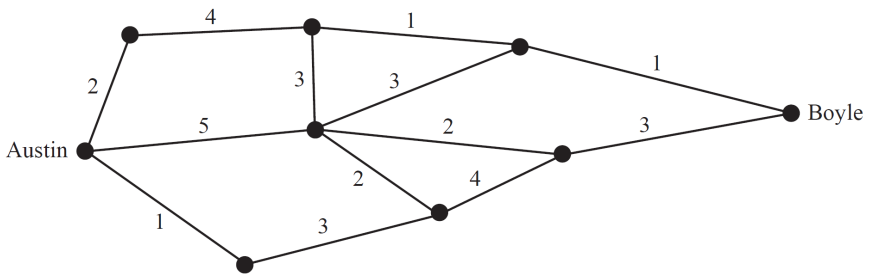
ANALYSIS -

- This topic area has typically been associated with low to mid-band style questions from our research.
- Implementing *Djikstra's algorithm* to solve questions in this topic has been a focus. This solution method is provided in detail in most worked solutions, reflecting its importance in NESA's *Networks Topic Guide*.
- Questions can require students to find a "shortest distance" by providing either a network diagram or a table, both of which are covered in the database.

Questions

1. Networks, FUR1 2009 VCE 2 MC

   The network shows the distances, in kilometres, along roads that connect the cities of Austin and Boyle.



   The shortest distance, in kilometres, from Austin to Boyle is

   A. 7

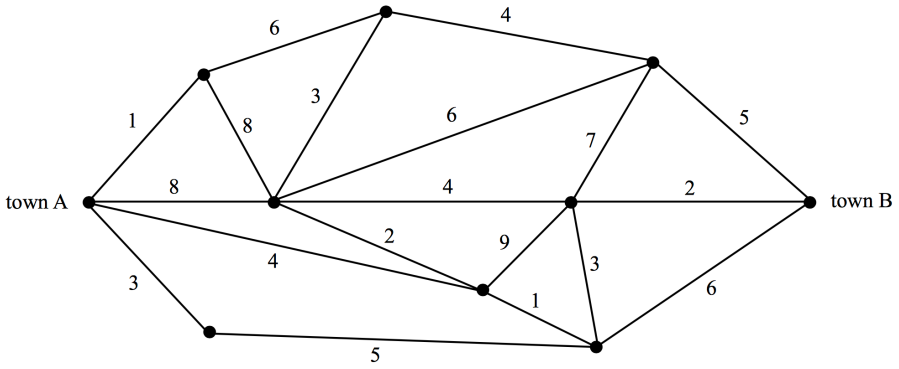
   B. 8

   C. 9

   D. 10

2. Networks, FUR1 2007 VCE 4 MC

The following network shows the distances, in kilometres, along a series of roads that connec A to town B.



The shortest distance, in kilometres, to travel from town A to town B is

A. 9

B. 10

C. 11

D. 12

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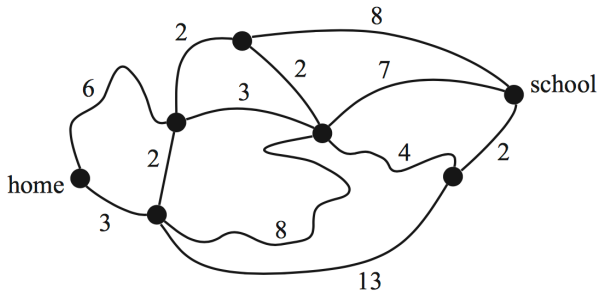
3. Networks, FUR1 2014 VCE 3 MC

The diagram below shows the network of roads that Stephanie can use to travel between home and school.



The numbers on the roads show the time, in minutes, that it takes her to ride a bicycle along each road.

Using this network of roads, the shortest time that it will take Stephanie to ride her bicycle from home to school is

A. 12 minutes

B. 13 minutes

C. 14 minutes

D. 15 minutes

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4. Networks, STD2 N2 SM-Bank 25 MC

In a town, there are four cafes W, X, Y and Z. The table shows the distances, in metres, of paved footpath connecting the cafes.

	W	X	Y	Z
W	–	150	–	120
X	150	–	140	70
Y	–	140	–	100
Z	120	70	100	–

A coffee supplier needs to visit each cafe.

What is the shortest distance she needs to walk along the paved footpath if she starts at cafe W?

A. 260 m

B. 320 m

C. 330 m

D. 360 m

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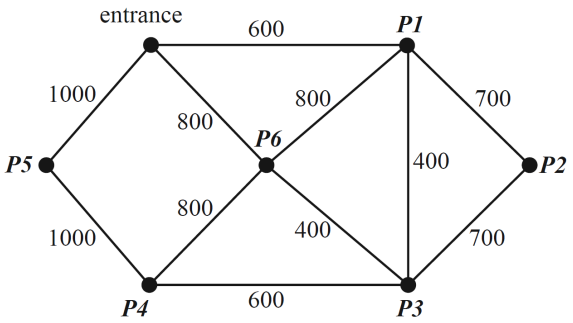
5. Networks, FUR2 2013 VCE 1

The vertices in the network diagram below show the entrance to a wildlife park and six picnic areas in the park: $P1$, $P2$, $P3$, $P4$, $P5$ and $P6$.

The numbers on the edges represent the lengths, in metres, of the roads joining these locations.



a. In this graph, what is the degree of the vertex at the entrance to the wildlife park? *(1 mark)*

b. What is the shortest distance, in metres, from the entrance to picnic area $P3$? *(1 mark)*

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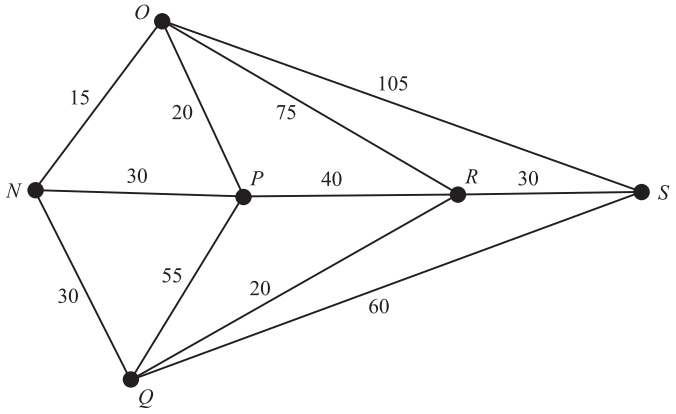
6. Networks, FUR2 2017 VCE 1

Bus routes connect six towns.

The towns are Northend (N), Opera (O), Palmer (P), Quigley (Q), Rosebush (R) and Seatown (S).

The graph below gives the cost, in dollars, of bus travel along these routes.

Bai lives in Northend (N) and he will travel by bus to take a holiday in Seatown (S).



a. Bai considers travelling by bus along the route Northend (N) – Opera (O) – Seatown (S). How much would Bai have to pay? *(1 mark)*

b. If Bai takes the cheapest route from Northend (N) to Seatown (S), which other town(s) will he pass through? *(1 mark)*

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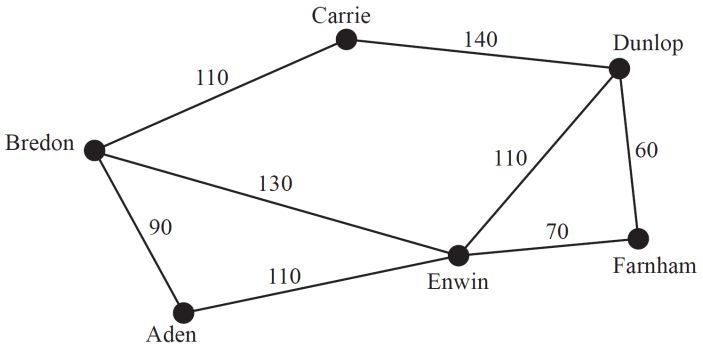
7. Networks, STD1 2011 VCE 1

Aden, Bredon, Carrie, Dunlop, Enwin and Farnham are six towns.

The network shows the road connections and distances between these towns in kilometres.



a. In kilometres, what is the shortest distance between Farnham and Carrie? ***(1 mark)***

b. How many different ways are there to travel from Farnham to Carrie without passing through any town more than once? ***(1 mark)***

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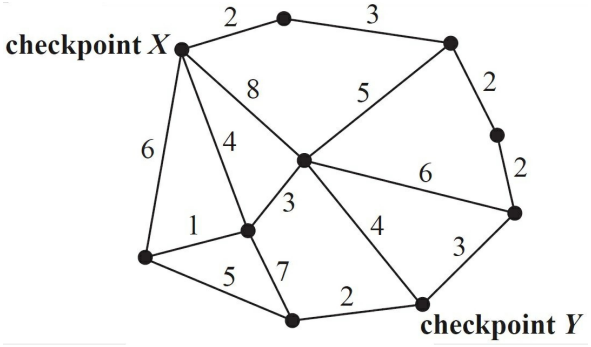
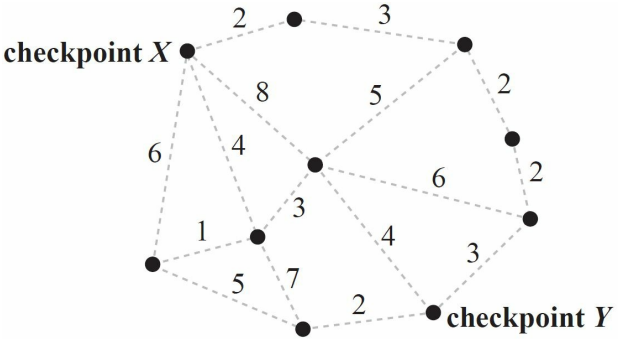
8. Networks, FUR2 2010 VCE 2

The diagram below shows a network of tracks (represented by edges) between checkpoints (represented by vertices) in a short-distance running course. The numbers on the edges indicate the time, in minutes, a team would take to run along each track.



A challenge requires teams to run from checkpoint X to checkpoint Y using these tracks.

What would be the shortest possible time for a team to run from checkpoint X to checkpoint Y? Mark the shortest route on the diagram below. ***(2 marks)***



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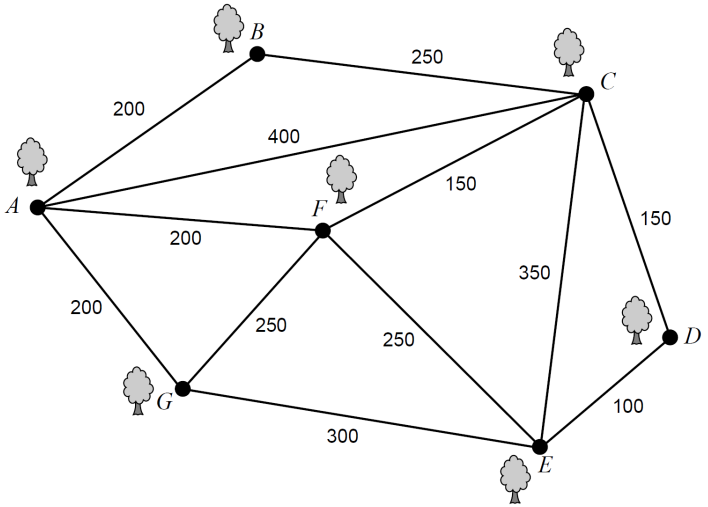
9. Networks, STD2 N2 SM-Bank 13

An estate has large open parklands that contain seven large trees.

The trees are denoted as vertices *A* to *G* on the network diagram below.

Walking paths link the trees as shown.

The numbers on the edges represent the lengths of the paths in metres.



Jamie is standing at *A* and Michelle is standing at *D*.

Write down the shortest route that Jamie can take and the distance travelled to meet Michelle at *D*. ***(1 mark)***

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10. Networks, STD2 N2 SM-Bank 28

In central Queensland, there are four petrol stations $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$. The table shows the length, in kilometres, of roads connecting these petrol stations.

	A	*B*	*C*	*D*
A	–	170	–	150
B	170	–	160	90
C	–	160	–	120
D	150	90	120	–

a. Construct a network diagram to represent the information in the table. ***(2 marks)***

b. A petrol tanker needs to refill each station. It starts at Station $\boldsymbol{A}$ and visits each station.
Calculate the shortest distance that can be travelled by the petrol tanker. In your answer, include the order the petrol stations are refilled. ***(2 marks)***

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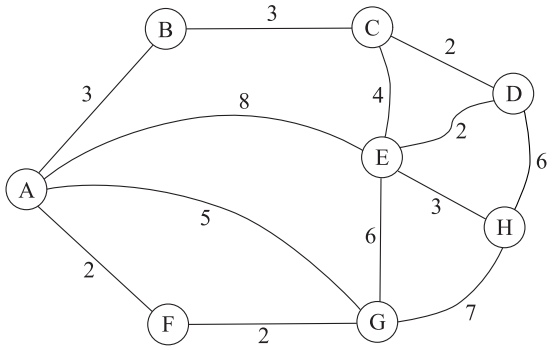
11. Networks, STD2 N2 SM-Bank 05

A network of roads is pictured below, with the distances of each road represented, in kilometres, on each edge.



A driver wants to travel from A to H in the shortest distance possible.

Describe the possible paths she can take, and the total distance she must travel. ***(2 marks)***

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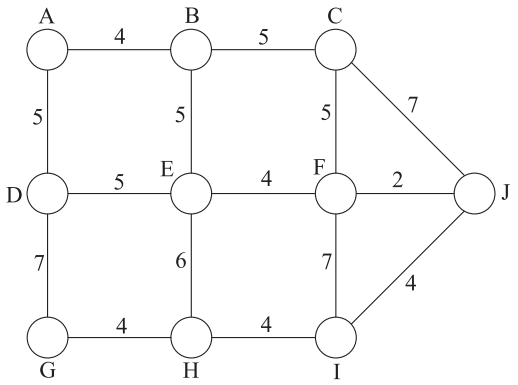
12. Networks, STD2 N2 SM-Bank 06



Describe the shortest path between A and J in the network above and its weight. ***(2 marks)***

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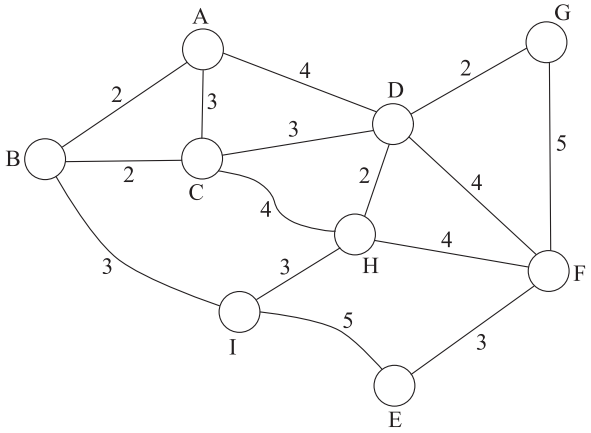
13. Networks, STD2 N2 SM-Bank 10

Consider the network pictured below.



Find the length of the shortest path from A to E. ***(2 marks)***

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14. Networks, STD2 N2 SM-Bank 12

The following table shows the travelling time, in minutes, between towns which are directly connected by roads.

A dash indicates that towns are not directly connected.

	A	*B*	*C*	*D*	*E*
A	0	50	18	10	—
B	50	0	28	32	20
C	18	28	0	—	50
D	10	32	—	0	80
E	—	20	50	80	0

(i) Draw a network diagram showing the information in this table. ***(1 mark)***

(ii) What is the shortest travelling time between *A* and *E*? ***(1 mark)***

Worked Solutions

1. Networks, FUR1 2009 VCE 2 MC

The shortest distance

$= 2 + 4 + 1 + 1$

$= 8$

$\Rightarrow B$

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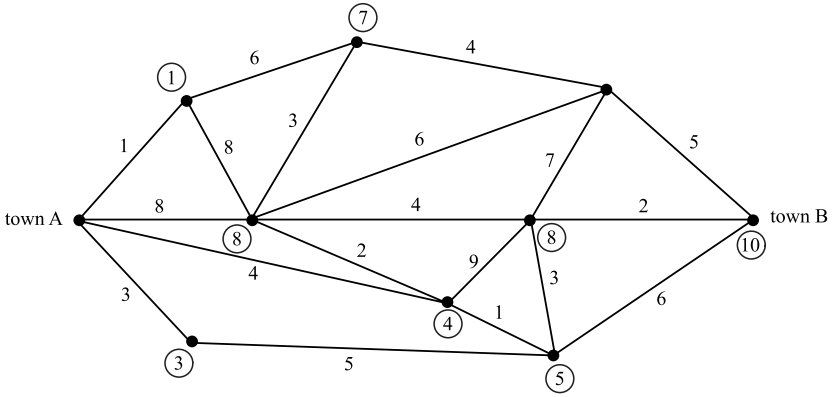
2. Networks, FUR1 2007 VCE 4 MC

Using Djikstra's algorithm:



Shortest path

$= 4 + 1 + 3 + 2$

$= 10$

$\Rightarrow B$

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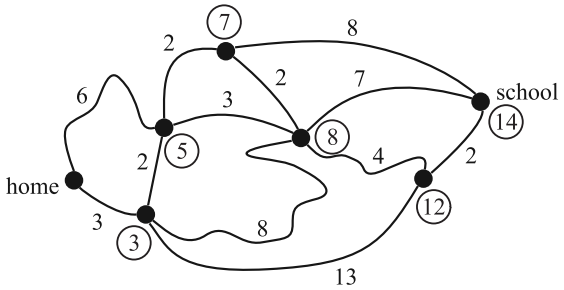
3. Networks, FUR1 2014 VCE 3 MC

Using Djikstra's algorithm:



Shortest time riding

$= 3 + 2 + 3 + 4 + 2$

$= 14$ minutes

$\Rightarrow C$

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4. Networks, STD2 N2 SM-Bank 25 MC

Possible paths:

$W - Z - X - Y = 120 + 70 + 140 = 330$ m

$W - Z - Y - X = 120 + 100 + 140 = 360$ m

$W - X - Z - Y = 150 + 70 + 100 = 320$ m

$W - X - Y - Z = 150 + 140 + 100 = 390$ m

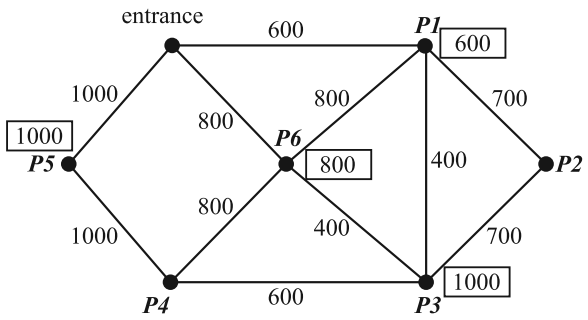
$\Rightarrow$ B

Worked Solutions

5. Networks, FUR2 2013 VCE 1

a. 3

b. Using Djikstra's algorithm:



Shortest distance

$= E - P1 - P3$

$= 600 + 400$

$= 1000$ m

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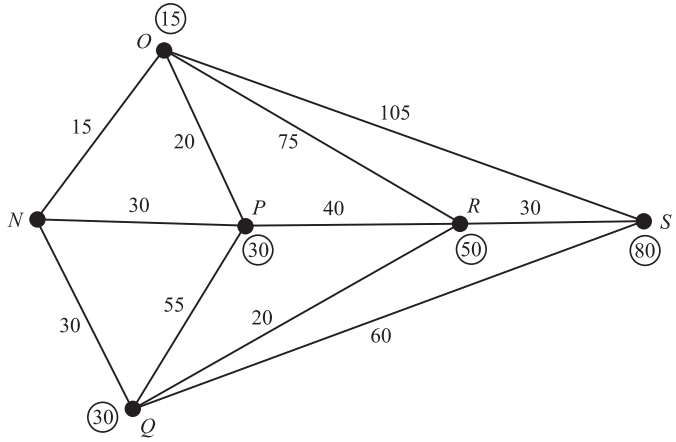
6. Networks, FUR2 2017 VCE 1

a. $\text{Cost} = 15 + 105$

$= \$120$

b. Using Djikstra's algorithm:



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7. Networks, STD1 2011 VCE 1

a. Farnham to Carrie (shortest)

$= 60 + 140$

$= 200$ km

b. Different paths are

$FDC, FEDC, FEBC,$

$FEABC, FDEBC,$

$FDEABC$

$\therefore$ 6 different ways

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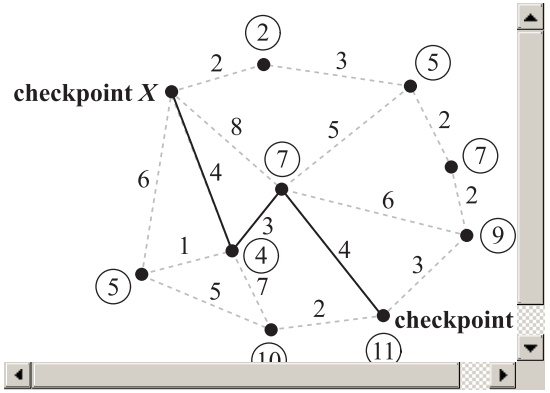
8. Networks, FUR2 2010 VCE 2

Using Djikstra's Algorithm:



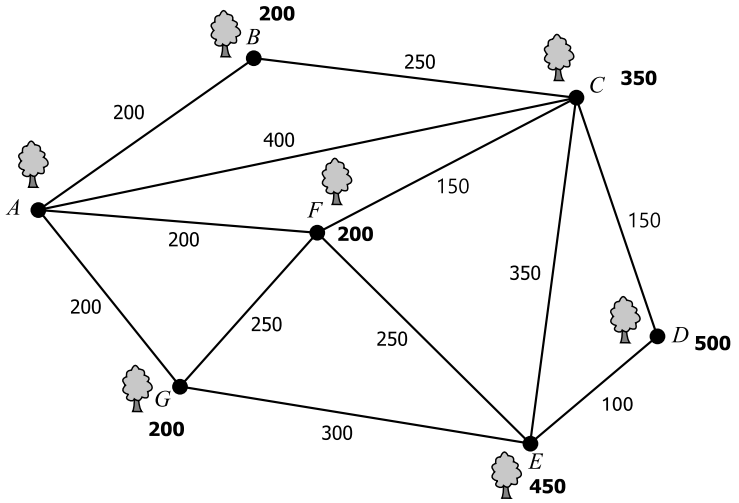
Shortest distance X to Y

$= 4 + 3 + 4$

$= 11$ minutes

9. Networks, STD2 N2 SM-Bank 13

One strategy - using Dijkstra's algorithm:



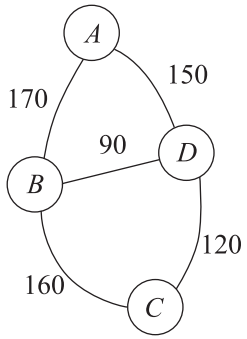
Shortest path is $AFCD$

Shortest distance $= 200 + 150 + 150$

$= 500$ metres

10. Networks, STD2 N2 SM-Bank 28

a.



b. Shortest Path from A (visiting all stations)

$A - B - D - C$

Distance $= 170 + 90 + 120$

$= 380$ km

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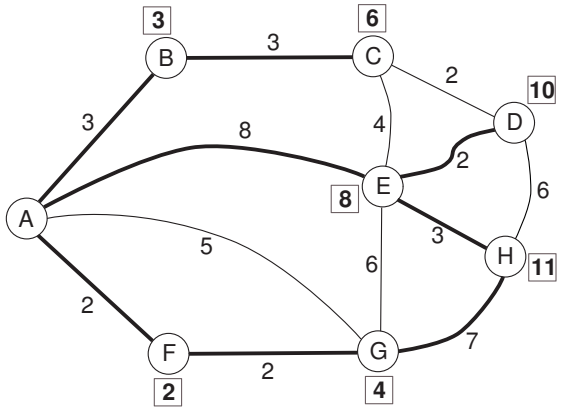
11. Networks, STD2 N2 SM-Bank 05

One Strategy: Dijkstra's algorithm



Shortest distance paths:

A - E - H or A - F - G - H

Shortest distance is 11 km.

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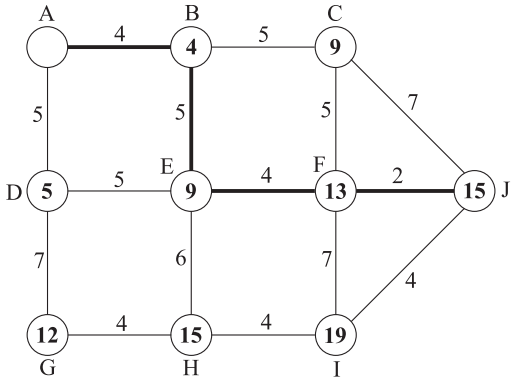
12. Networks, STD2 N2 SM-Bank 06

One Strategy: Using Dijkstra's algorithm



Shortest path is:

A - B - E - F - J

$\therefore$ **Weight of shortest path**

$= 4 + 5 + 4 + 2$

$= 15$

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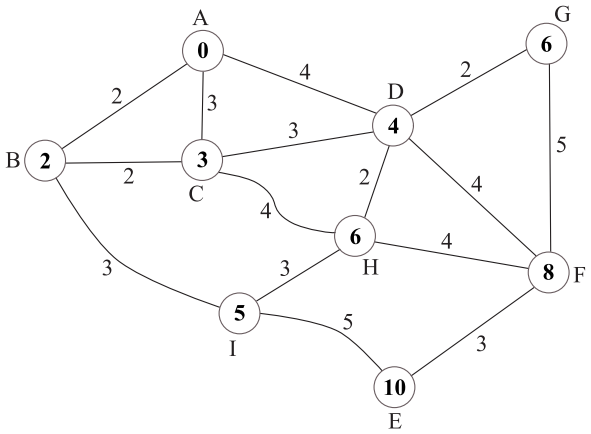
13. Networks, STD2 N2 SM-Bank 10

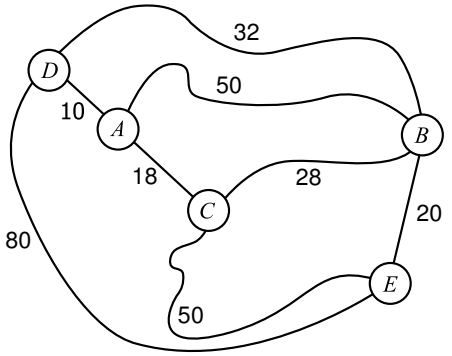
One strategy - Using Dijkstra's algorithm:



The shortest path A - B - I - E has a distance (weight) of 10.

14. Networks, STD2 N2 SM-Bank 12

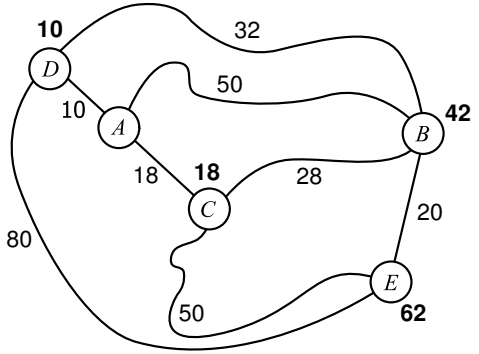
(i)



NOT TO SCALE

Important to note that network diagrams do not need to be drawn to scale.

(ii) **One strategy - using Dijkstra's algorithm:**



NOT TO SCALE

$\therefore$ **Shortest travelling time is the path** $A - D - B - E$

$= 10 + 32 + 20$

$= 62$ **minutes**