STD 1: Statistical Analysis (Std 1), S2 Relative Frequency and Probability (Y11)

Combinations and Single Stage Events (Std 1)

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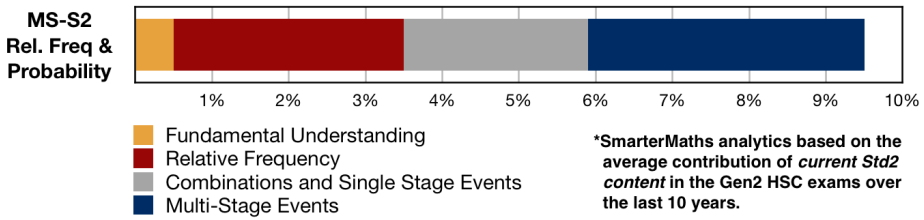
Exam Equivalent Time: 54 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

General 2 Exam Contribution History
S2 Relative Frequency and Probability



***SmarterMaths analytics based on the average contribution of *current Std2 content* in the Gen2 HSC exams over the last 10 years.**

IMPORTANT FEATURES AND TIPS FROM 2UG EXAM HISTORY

- *MS-S2 Relative Frequency and Probability* was a major contributor to the old Gen2 course, contributing an average of 9.5% per exam over the past decade (note past allocations are no guarantee of future contributions but can nonetheless cast light on a topic's likely importance).
- This analysis looks at the sub-topics of *Combinations and Single-Stage Events (2.4%)*.

ANALYSIS - What to Expect and Common pitfalls

- *Combinations* have been examined at least once every year in the Gen2 exam, often involving questions on number plates, PINs, ice-cream flavours, etc...
- This topic area is typically poorly answered, with sub-50% mean marks common.
- *Unordered selection* questions that require factorial notation to solve are regarded as beyond the scope of Standard 1 and the database reflects this.
- With mean marks regularly below 50% in past Gen2 exams, this sub-topic is at the very high difficulty end of possible Standard 1 content.
- *Single-Stage events* are a simpler version of the much more common multi-stage event probability.

Questions

1. Probability, 2UG 2011 HSC 5 MC

   The letters A, B and C are used to make a three-letter company name. Each letter is used only once.

   How many different company names can be made?

   (A) 3

   (B) 6

   (C) 9

   (D) 27

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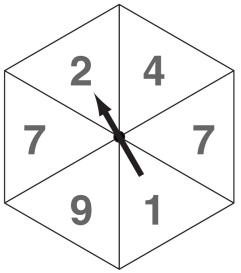
2. Probability, 2UG 2005 HSC 11 MC

   The diagram shows a spinner.

   The arrow is spun and will stop in one of the six sections.

   What is the probability that the arrow will stop in a section containing a number greater than 4?

   (A) $\frac{2}{5}$

   (B) $\frac{2}{3}$

   (C) $\frac{1}{3}$

   (D) $\frac{1}{2}$

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3. Probability, 2UG 2014 HSC 6 MC

A cafe menu has 3 entrees, 5 main courses and 2 desserts. Ariana is choosing a three-course meal consisting of an entree, a main course and a dessert.

How many different three-course meals can Ariana choose?

(A) 3

(B) 10

(C) 15

(D) 30

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4. Probability, 2UG 2016 HSC 6 MC

RAP Data - Bottom 15%: School result (92%) was 3% above state average (89%)

Ben has 6 shirts, 4 ties and 2 jackets.

How many different outfits of a shirt, a tie and a jacket can he choose?

(A) 12

(B) 16

(C) 24

(D) 48

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5. Probability, 2UG 2013 HSC 1 MC

Which of the following events would be LEAST likely to occur?

(A) Tossing a fair coin and obtaining a head

(B) Rolling a standard six-sided die and obtaining a 3

(C) Randomly selecting the letter 'G' from the 26 letters of the alphabet

(D) Winning first prize in a raffle of 100 tickets in which you have 4 tickets

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6. Probability, 2UG 2008 HSC 16 MC

A bag contains some marbles. The probability of selecting a blue marble at random from this bag is $\frac{3}{8}$.

Which of the following could describe the marbles that are in the bag?

(A) 3 blue, 8 red

(B) 6 blue, 11 red

(C) 3 blue, 4 red, 4 green

(D) 6 blue, 5 red, 5 green

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7. Probability, 2UG 2010 HSC 8 MC

RAP Data - Bottom 12%: School result (55%) was 2% above state average (53%)

A bag contains red, green, yellow and blue balls.

Colour	*Probability*
Red	$\frac{1}{3}$
Green	$\frac{1}{4}$
Yellow	?
Blue	$\frac{1}{6}$

The table shows the probability of choosing a red, green, or blue ball from the bag.

If there are 12 yellow balls in the bag, how many balls are in the bag altogether

(A) 16

(B) 36

(C) 48

(D) 60

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8. Probability, 2UG 2008 HSC 18 MC

New car registration plates contain two letters followed by two numerals followed by two more letters eg AC 12 DC. Letters and numerals may be repeated.

Which of the following expressions gives the correct number of car registration plates that begin with the letter M?

(A) $26^3 \times 10^2$

(B) $25^3 \times 10^2$

(C) $26^4 \times 10^2$

(D) $25^4 \times 10^2$

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9. Probability, 2UG 2015 HSC 21 MC

Four cards, labelled 2, 3, 5 and 7, are placed on a table to form a four-digit number.

How many different numbers greater than 3000 can be formed?

(A) 6

(B) 12

(C) 18

(D) 24

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10. Probability, 2UG 2017 HSC 15 MC

The faces on a twenty-sided die are labelled \$0.05, \$0.10, \$0.15, … , \$1.00.

The die is rolled once.

What is the probability that the amount showing on the upper face is more than 50 cents but less than 80 cents?

A. $\frac{1}{4}$

B. $\frac{3}{10}$

C. $\frac{7}{20}$

D. $\frac{1}{2}$

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11. Probability, 2UG 2005 HSC 20 MC

Dave's school has computer security codes made up of four digits (eg 0773). Juanita's school has computer security codes made up of five digits (eg 30568).

How many more codes are available at Juanita's school than at Dave's school?

(A) 10

(B) 50

(C) 90 000

(D) 100 000

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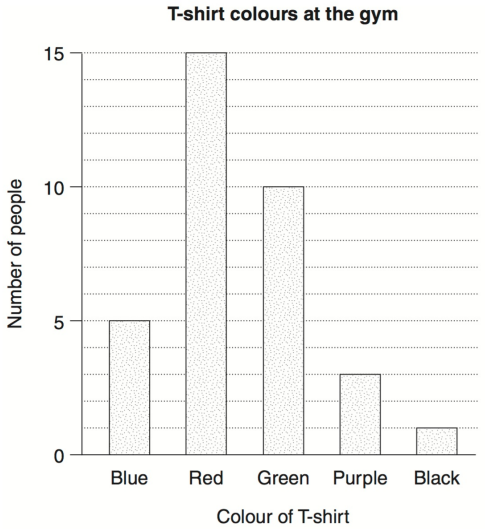
12. Probability, 2UG 2010 HSC 23c

Part i: RAP Data - Bottom 11%: School result (96%) was 2% above state average (94%)

On Saturday, Jonty recorded the colour of T-shirts worn by the people at his gym. The results are shown in the graph.



(i) How many people were at the gym on Saturday? (Assume everyone was wearing a T-shirt). ***(1 mark)***

(ii) What is the probability that a person selected at random at the gym on Saturday, would be wearing either a blue or green T-shirt? ***(1 mark)***

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13. Probability, 2UG 2013 HSC 29c

Mary is designing a website that requires unique logins to be generated.

She plans to generate the logins using two capital letters from the alphabet followed by a series of numerals from 0 to 9 inclusive. All logins will have the same number of numerals. Repetition of letters and numerals is allowed.

What is the minimum number of numerals required for each login so that Mary can generate at least 3 million logins?

Justify your answer with suitable calculations. ***(2 marks)***

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14. Probability, 2UG 2005 HSC 23a

There are **100** tickets sold in a raffle. Justine sold all **100** tickets to five of her friends. The number of tickets she sold to each friend is shown in the table.

Friend	*Number of tickets*
Danielle	45
Khalid	5
Nancy	10
Shani	14
Herman	26
Total	100

(i) Justine claims that each of her friends is equally likely to win first prize. Give a reason why Justine's statement is NOT correct. ***(1 mark)***

(ii) What is the probability that first prize is NOT won by Khalid or Herman? ***(2 marks)***

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15. Probability, 2UG 2015 HSC 26e

Part i: RAP Data - Bottom 25%: School result (92%) was 8% above state average (84%)

The table shows the relative frequency of selecting each of the different coloured jelly beans from packets containing green, yellow, black, red and white jelly beans.

Colour	*Relative frequency*
Green	0.32
Yellow	0.13
Black	0.14
Red	
White	0.24

i. What is the relative frequency of selecting a red jelly bean? ***(1 mark)***

ii. Based on this table of relative frequencies, what is the probability of NOT selecting a black jelly bean? ***(1 mark)***

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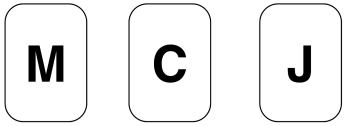
16. Probability, 2UG 2006 HSC 25a

Three cards labelled C, J and M can be arranged in any order.

eg. 

(i) In how many different ways can the cards be arranged? ***(1 mark)***

(ii) What is the probability that the second card in an arrangement is a J? ***(1 mark)***

(iii) What is the probability that the last card in an arrangement is not a C? ***(1 mark)***

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17. Probability, 2UG 2010 HSC 26a

Part ii: RAP Data - Bottom 19%: School result (35%) was 5% above state average (30%)

A design of numberplates has a two-digit number, two letters and then another two-digit number, for example

22 AC 14 or 76 BB 08.

(i) How many different numberplates are possible using this design? ***(1 mark)***

(ii) Jo's birthday is 30 December 1992, so she would like a numberplate with either

30 JO 12 or 19 JO 92

Jo can order a numberplate with 'JO' in the middle but will have to have randomly selected numbers on either side.

What is the probability that Jo is issued with one of the numberplates she would like? ***(2 marks)***

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18. Probability, 2UG 2012 HSC 26a

Part ii: RAP Data - Bottom 15%: School result (45%) was 4% above state average (41%)

Postcodes in Australia are made up of four digits eg 2040.

(i) How many different postcodes beginning with a 2 are possible? ***(1 mark)***

(ii) Peta remembers that the first two digits of a town's postcode are **2** and then 4. She is unable to remember the rest of the postcode. ***(1 mark)***

2	4	?	?

What is the probability that Peta guesses the correct postcode?

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19. Probability, 2UG 2018 HSC 26f

A toy shop sells buckets and spades separately. Buckets are available in one of six colours. Spades are also available in one of the same six colours.

Abdul wants to buy a bucket-and-spade set where the bucket and spade are of different colours.

How many different bucket-and-spade sets are possible for Abdul to buy? ***(1 mark)***

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20. Probability, 2UG 2008 HSC 24b

Part iv: RAP Data - Bottom 7%: School result (10%) was equal to state average (10%)

Three-digit numbers are formed from five cards labelled **1**, **2**, **3**, **4** and **5**.

(i) How many different three-digit numbers can be formed? ***(1 mark)***

(ii) If one of these numbers is selected at random, what is the probability that it is odd? ***(1 mark)***

(iii) How many of these three-digit numbers are even? ***(1 mark)***

(iv) What is the probability of randomly selecting a three-digit number less than **500** with its digits arranged in descending order? ***(2 marks)***

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21. Probability, 2UG 2009 HSC 23b

A personal identification number (PIN) is made up of four digits. An example of a PIN is

0229.

(i) When all ten digits are available for use, how many different PINs are possible? ***(1 mark)***

(ii) Rhys has forgotten his four-digit PIN, but knows that the first digit is either 5 or 6.

What is the probability that Rhys will correctly guess his PIN in one attempt? ***(1 mark)***

Worked Solutions

1. Probability, 2UG 2011 HSC 5 MC

$$\begin{aligned}\#\text{ Outcomes} &= 3 \times 2 \times 1 \\ &= 6 \\ \Rightarrow B\end{aligned}$$

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2. Probability, 2UG 2005 HSC 11 MC

$$\begin{aligned}&P(\text{number greater than 4}) \\ &= P(7) + P(9) \\ &= \frac{2}{6} + \frac{1}{6} \\ &= \frac{1}{2} \\ \Rightarrow\ &D\end{aligned}$$

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3. Probability, 2UG 2014 HSC 6 MC

$$\begin{aligned}\#\text{ Combinations} &= 3 \times 5 \times 2 \\ &= 30 \\ \Rightarrow\ &D\end{aligned}$$

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4. Probability, 2UG 2016 HSC 6 MC

$$\begin{aligned}&\text{Combinations} \\ &= 6 \times 4 \times 2 \\ &= 48 \\ \Rightarrow\ &D\end{aligned}$$

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5. Probability, 2UG 2013 HSC 1 MC

$$P(A) = \frac{1}{2},\ P(B) = \frac{1}{6}$$

$$P(C) = \frac{1}{26},\ P(D) = \frac{4}{100} = \frac{1}{25}$$

$\Rightarrow C$ is the least likely.

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6. Probability, 2UG 2008 HSC 16 MC

$$P(B) = \frac{3}{8}$$

$$\text{In } A,\ P(B) = \frac{3}{11}$$

$$\text{In } B,\ P(B) = \frac{6}{18} = \frac{1}{3}$$

$$\text{In } C,\ P(B) = \frac{3}{11}$$

$$\text{In } D,\ P(B) = \frac{6}{16} = \frac{3}{8}$$

$\Rightarrow D$

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7. Probability, 2UG 2010 HSC 8 MC

$$\begin{aligned}P(R) + P(G) + P(Y) + P(B) &= 1 \\ \frac{1}{3} + \frac{1}{4} + P(Y) + \frac{1}{6} &= 1 \\ P(Y) &= 1 - \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{6}\right) \\ &= 1 - \frac{9}{12} \\ &= \frac{1}{4}\end{aligned}$$

$$P(Y) = \frac{\#\text{ Yellow balls}}{\#\text{ Total balls}}$$

$$\frac{1}{4} = \frac{12}{\#\text{ Total balls}}$$

$\therefore\ \#\text{ Total balls} = 48$

$\Rightarrow C$

Worked Solutions

8. Probability, 2UG 2008 HSC 18 MC

$$\begin{aligned}
&\#\text{ Plates beginning with M} \\
&\quad = 1 \times 26 \times 10 \times 10 \times 26 \times 26 \\
&\quad = 26^3 \times 10^2 \\
&\Rightarrow A
\end{aligned}$$

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9. Probability, 2UG 2015 HSC 21 MC

$$\begin{aligned}
&\#\text{ Combinations greater than 3000} \\
&\quad = 3 \times 3 \times 2 \times 1 \\
&\quad = 18 \\
&\Rightarrow C
\end{aligned}$$

♦ *Mean mark 48%.*

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10. Probability, 2UG 2017 HSC 15 MC

Possible faces that satisfy are:

55c, 60c, 65c, 70c, 75c

$$\begin{aligned}
\therefore \text{ Probability} &= \frac{5}{20} \\
&= \frac{1}{4} \\
&\Rightarrow A
\end{aligned}$$

♦ *Mean mark 50%.*

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11. Probability, 2UG 2005 HSC 20 MC

$$\begin{aligned}
&\#\text{ Codes at Dave's school} \\
&\quad = 10 \times 10 \times 10 \times 10 \\
&\quad = 10\,000 \\
&\#\text{ Codes at Juanita's school} \\
&\quad = 10^5 \\
&\quad = 100\,000 \\
&\therefore \text{ Extra Codes} = 100\,000 - 10\,000 \\
&\qquad\qquad = 90\,000 \\
&\Rightarrow C
\end{aligned}$$

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12. Probability, 2UG 2010 HSC 23c

(i) $$\begin{aligned}
\#\text{ People} &= 5 + 15 + 10 + 3 + 1 \\
&= 34
\end{aligned}$$

(ii) $$\begin{aligned}
P(B \text{ or } G) &= P(B) + P(G) \\
&= \frac{5}{34} + \frac{10}{34} \\
&= \frac{15}{34}
\end{aligned}$$

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13. Probability, 2UG 2013 HSC 29c

$$\begin{aligned}
&\#\text{ Combinations must be} > 3 \text{ million:} \\
&\text{If we have 3 numerals,} \\
&\quad \#\text{ Combinations} = 26 \times 26 \times 10 \times 10 \times 10 \\
&\qquad\qquad = 676\,000 < 3\,000\,000 \\
&\quad \Rightarrow \text{need more numeral(s)} \\
&\text{If we have 4 numerals,} \\
&\quad \#\text{ Combinations} = 26 \times 26 \times 10 \times 10 \times 10 \times 10 \\
&\qquad\qquad = 6760\,000 > 3\,000\,000 \\
&\therefore \text{ Minimum number of numerals} = 4
\end{aligned}$$

♦ *Mean mark 34%*
COMMENT: Students can use their rough working to find an appropriate "number of numerals" where their answer should start.

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14. Probability, 2UG 2005 HSC 23a

(i) The claim is incorrect because each of her friends bought a different number of tickets and therefore their chances of winning are different.

(ii) Number of tickets not sold to K or H

$$= 45 + 10 + 14$$
$$= 69$$

$\therefore$ Probability 1st prize NOT won by K or H

$$= \frac{69}{100}$$

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15. Probability, 2UG 2015 HSC 26e

i. Relative frequency of red

$$= 1 - (0.32 + 0.13 + 0.14 + 0.24)$$
$$= 1 - 0.83$$
$$= 0.17$$

ii. $P(\text{not selecting black})$

$$= 1 - P(\text{selecting black})$$
$$= 1 - 0.14$$
$$= 0.86$$

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16. Probability, 2UG 2006 HSC 25a

(i) $\#\text{ Arrangements} = 3 \times 2 \times 1$

$$= 6$$

(ii) $P(\text{second card is J})$

$$= \frac{1}{3}$$

(iii) $P(\text{last card is not a C})$

$$= 1 - P(\text{last card is a C})$$
$$= 1 - \frac{1}{3}$$
$$= \frac{2}{3}$$

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17. Probability, 2UG 2010 HSC 26a

(i) $\#\text{ Combinations} = 10 \times 10 \times 26 \times 26 \times 10 \times 10$

$$= 6\,760\,000$$

♦ *Mean mark 41%*

(ii) $\#\text{ Possible numberplates}$

$$= 10 \times 10 \times 10 \times 10$$
$$= 10\,000$$

$\therefore P(30\text{ JO }12) + P(19\text{ JO }92)$

$$= \frac{1}{10\,000} + \frac{1}{10\,000}$$
$$= \frac{1}{5000}$$

♦♦ *Mean mark 30%*
IMPORTANT: Since the middle letters of "JO" can be guaranteed, the focus becomes purely on the 4 surrounding digits.

18. Probability, 2UG 2012 HSC 26a

(i) Different postcodes begining with 2

$$\begin{aligned} &= 1 \times 10 \times 10 \times 10 \\ &= 1000 \end{aligned}$$

♦ *Mean marks of 43% and 41% for parts (i) and (ii) respectively.*

(ii) Number of postcodes beginning with 2, 4

$$\begin{aligned} &= 1 \times 1 \times 10 \times 10 \\ &= 100 \end{aligned}$$

$$\therefore P\,(\text{Correct}) = \frac{1}{100}$$

19. Probability, 2UG 2018 HSC 26f

♦ *Mean mark 35%.*

$$\begin{aligned} \text{Total combinations} &= \text{ bucket choices} \times \text{spade choices} \\ &= 6 \times 5 \\ &= 30 \end{aligned}$$

20. Probability, 2UG 2008 HSC 24b

(i) # Different numbers

$$\begin{aligned} &= 5 \times 4 \times 3 \\ &= 60 \end{aligned}$$

♦ *Mean mark 45%.*

(ii) The last digit must be one of the 5 numbers, of which 3 are odd

$$\therefore \text{P(odd)} = \frac{3}{5}$$

(iii) $\text{P(even)} = 1 - \text{P(odd)} = \frac{2}{5}$

$$\begin{aligned} \therefore \text{ Number of even numbers} &= \frac{2}{5} \times 60 \\ &= 24 \end{aligned}$$

♦ *Mean mark 48%.*

(iv) The numbers that satisfy the criteria:

432, 431, 421, 321

$$\therefore \text{P(selection)} = \frac{4}{60} = \frac{1}{15}$$

♦♦♦ *Mean mark 10%.*

21. Probability, 2UG 2009 HSC 23b

(i)

$$\begin{aligned} \#\text{ Combinations} &= 10 \times 10 \times 10 \times 10 \\ &= 10\,000 \end{aligned}$$

♦ *Mean mark 43%*

(ii)

$$\begin{aligned} \#\text{ Combinations} &= 2 \times 10 \times 10 \times 10 \\ &= 2000 \end{aligned}$$

$$\begin{aligned} P(\text{Correct PIN}) &= \frac{\#\text{ Correct PINS}}{\#\text{ Combinations}} \\ &= \frac{1}{2000} \end{aligned}$$

♦♦♦ *Mean mark 18%*
MARKER'S COMMENT: A common error is finding the number of possible combinations but not then calculating the probability.