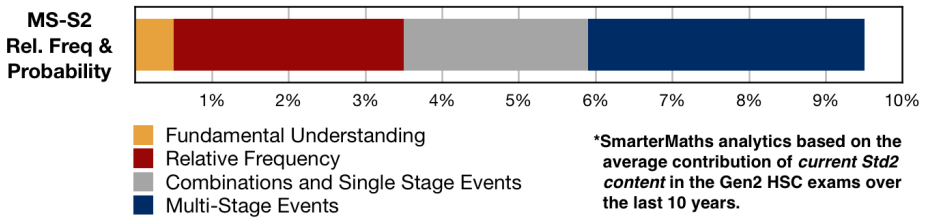
STD 1: Statistical Analysis (Std 1), S2 Relative Frequency and Probability (Y11)

Multi-Stage Events (Std 1)

Teacher: Kirtana Hariharan

Exam Equivalent Time: 78 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

General 2 Exam Contribution History
S2 Relative Frequency and Probability



***SmarterMaths analytics based on the average contribution of *current Std2 content* in the Gen2 HSC exams over the last 10 years.**

IMPORTANT FEATURES AND TIPS FROM 2UG EXAM HISTORY

- *MS-S2 Relative Frequency and Probability* was a major contributor to the old Gen2 course, contributing an average of 9.5% per exam over the past decade.
- This analysis looks at the sub-topic *Multi-Stage Events (3.6%).*

ANALYSIS - What to Expect and Common pitfalls

- *Multi-stage Events* has contributed two questions to a majority of HSC Gen2 papers over the last decade. Students have found this sub-topic very challenging and it represents the upper end of the difficulty scale for Standard 1 content.
- Most multi-part questions provide an opportunity for easier marks in the early section(s) which should be taken advantage of.
- Within this topic, *probability trees* have been responsible for two-thirds of the total mark allocation and have been examined in 5 of the last 6 years. This specific area, answered poorly in 2016 and 2015, should be carefully reviewed.
- Front of mind when looking at probability questions - examiners *regularly* provide opportunities for applying complementary probabilities, posing problems that can be more efficiently solved using the identity *"P(E) = 1 - P(Event NOT happening)".*

Questions

1. Probability, 2UG 2006 HSC 10 MC

   Kay randomly selected a marble from a bag of marbles, recorded its colour and returned it to the bag. She repeated this process a number of times.

Colour	*Tally*	*Frequency*
Red	𝍸 ||	7
Blue	|||	3
Yellow	||	2
Green	|||	4
Purple	𝍸 |||	8

   Based on these results, what is the best estimate of the probability that Kay will choose a green marble on her next selection?

   (A) $\frac{5}{24}$

   (B) $\frac{1}{24}$

   (C) $\frac{1}{6}$

   (D) $\frac{1}{5}$

2. Probability, 2UG 2007 HSC 10 MC

Each time she throws a dart, the probability that Mary hits the dartboard is $\frac{2}{7}$.

She throws two darts, one after the other.

What is the probability that she hits the dartboard with both darts?

(A) $\frac{1}{21}$

(B) $\frac{4}{49}$

(C) $\frac{2}{7}$

(D) $\frac{4}{7}$

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3. Probability, 2UG 2014 HSC 19 MC

Jaz has 2 bags of apples.

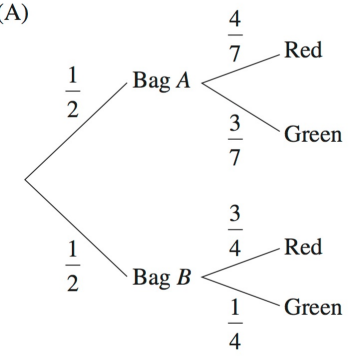
Bag A contains 4 red apples and 3 green apples.

Bag B contains 3 red apples and 1 green apple.

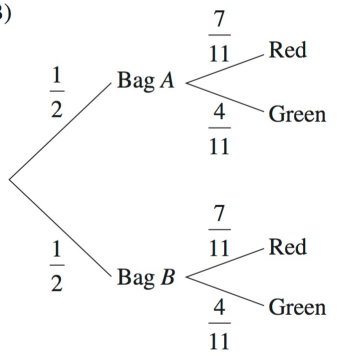
Jaz chooses an apple from one of the bags.

Which tree diagram could be used to determine the probability that Jaz chooses a red apple?

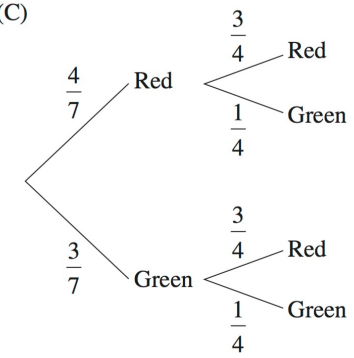
(A)



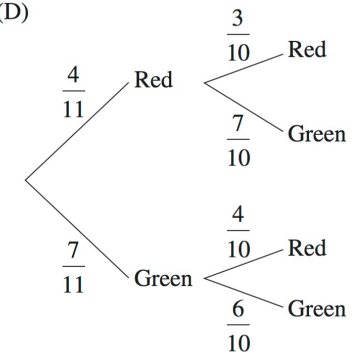
(B)



(C)



(D)



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4. Probability, 2UG 2015 HSC 16 MC

The probability of winning a game is $\frac{7}{10}$.

Which expression represents the probability of winning two consecutive games?

(A) $\frac{7}{10} \times \frac{6}{9}$

(B) $\frac{7}{10} \times \frac{6}{10}$

(C) $\frac{7}{10} \times \frac{7}{9}$

(D) $\frac{7}{10} \times \frac{7}{10}$

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5. Probability, 2UG 2012 HSC 12 MC

RAP Data - Bottom 15%: School result (29%) was 4% above state average (25%)

Two unbiased dice, each with faces numbered 1, 2, 3, 4, 5, 6, are rolled.

What is the probability of a 6 appearing on at least one of the dice?

(A) $\frac{1}{6}$

(B) $\frac{11}{36}$

(C) $\frac{25}{36}$

(D) $\frac{5}{6}$

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6. Probability, 2UG 2013 HSC 18 MC

Two unbiased dice, each with faces numbered 1, 2, 3, 4, 5, 6, are rolled.

What is the probability of obtaining a sum of 6?

(A) $\frac{1}{6}$

(B) $\frac{1}{12}$

(C) $\frac{5}{12}$

(D) $\frac{5}{36}$

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7. Probability, 2UG 2004 HSC 18 MC

Two dice are rolled. What is the probability that only one of the dice shows a six?

(A) $\frac{5}{36}$

(B) $\frac{1}{6}$

(C) $\frac{5}{18}$

(D) $\frac{11}{36}$

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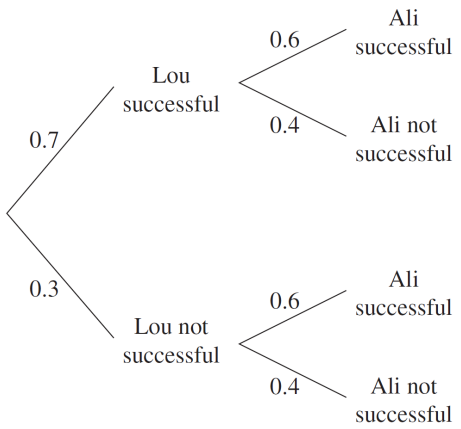
8. Probability, 2UG 2010 HSC 20 MC

Lou and Ali are on a fitness program for one month. The probability that Lou will finish the program successfully is 0.7 while the probability that Ali will finish successfully is 0.6. The probability tree shows this information



What is the probability that only one of them will be successful ?

(A) **0.18**

(B) **0.28**

(C) **0.42**

(D) **0.46**

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9. Probability, STD2 2011 HSC 26a

The two spinners shown are used in a game.

Each arrow is spun once. The score is the total of the two numbers shown by the arrows. A table is drawn up to show all scores that can be obtained in this game.

		Spinner B			
		1	**1**	**2**	**3**
Spinner A	**1**	2	2	3	4
	1	2	2	3	4
	3	4	4	X	6

(i) What is the value of X in the table? ***(1 mark)***

(ii) What is the probability of obtaining a score less than 4? ***(1 mark)***

(iii) On Spinner B, a 2 is obtained. What is the probability of obtaining a score of 3? ***(1 mark)***

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10. Probability, 2UG 2014 HSC 28c

A fair coin is tossed three times. Using a tree diagram, or otherwise, calculate the probability of obtaining two heads and a tail in any order. ***(2 marks)***

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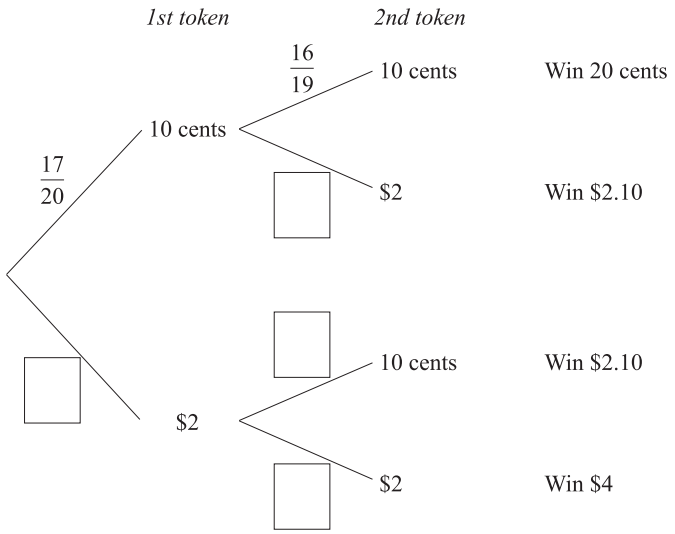
11. Probability, STD2 2018 HSC 30d

A game consists of two tokens being drawn at random from a barrel containing 20 tokens. There are 17 tokens labelled 10 cents and 3 tokens labelled $2. The player wins the total value of the two tokens drawn.

Complete the probability tree by writing the missing probabilities in the boxes. ***(2 marks)***

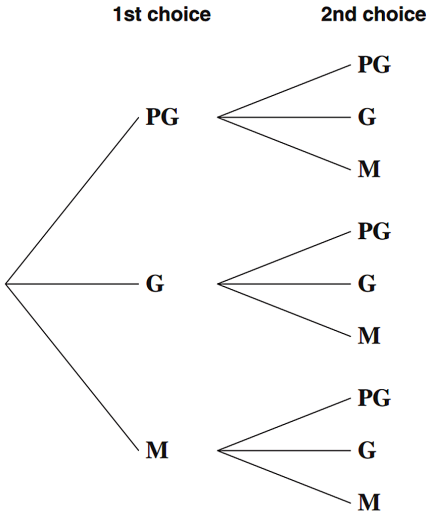


12. Probability, 2UG 2007 HSC 25c

In a stack of **10** DVDs, there are **5** rated **PG**, **3** rated **G** and **2** rated **M**.

(i) A DVD is selected at random. What is the probability that it is rated **M**? ***(1 mark)***

(ii) Grant chooses two DVDs at random from the stack. Copy or trace the tree diagram into your writing booklet. Complete your tree diagram by writing the correct probability on each branch. ***(2 marks)***



(iii) Calculate the probability that Grant chooses two DVDs with the same rating. ***(2 marks)***

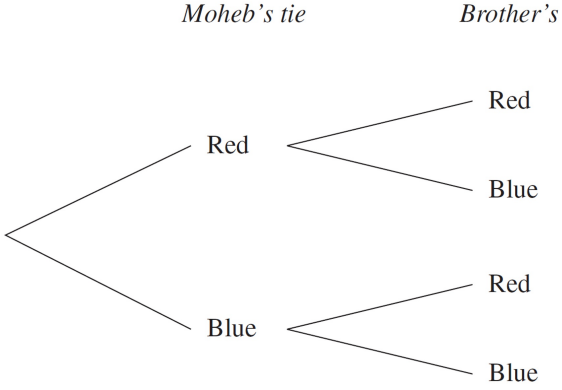
13. Probability, 2UG 2005 HSC 23c

Moheb owns five red and seven blue ties. He chooses a tie at random for himself and puts it on. He then chooses another tie at random, from the remaining ties, and gives it to his brother.

(i) What is the probability that Moheb chooses a red tie for himself? ***(1 mark)***

(ii) Copy the tree diagram into your writing booklet.

Complete your tree diagram by writing the correct probability on each branch. ***(2 marks)***



(iii) Calculate the probability that both of the ties are the same colour. ***(2 marks)***

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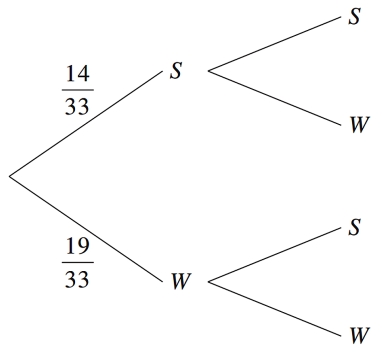
14. Probability, 2UG 2012 HSC 27e

A box contains 33 scarves made from two different fabrics. There are 14 scarves made from silk (S) and 19 made from wool (W).
Two girls each select, at random, a scarf to wear from the box.

(i) Copy and complete the probability tree diagram in your answer booklet. ***(2 marks)***



(ii) Calculate the probability that the two scarves selected are made from silk. ***(1 mark)***

(iii) Calculate the probability that the two scarves selected are made from different fabrics. ***(2 marks)***

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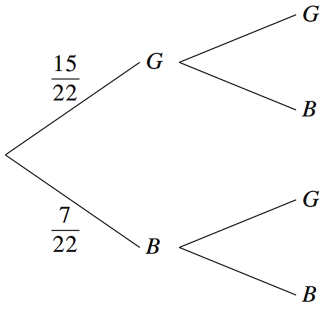
15. Probability, 2UG 2013 HSC 30b

In a class there are 15 girls *(G)* and 7 boys *(B)*. Two students are chosen at random to be class representatives.

(i) Copy and complete the tree diagram in your answer booklet. ***(2 marks)***



(ii) What is the probability that the two students chosen are of the same gender? ***(2 marks)***

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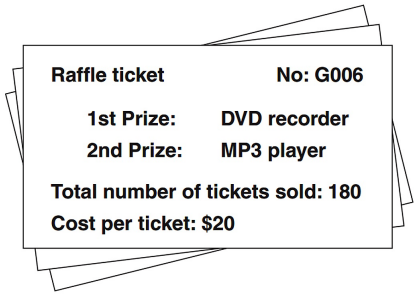
16. Probability, 2UG 2006 HSC 25c

Sonia buys three raffle tickets.



(i) What is the probability that Sonia wins first prize? ***(1 mark)***

(ii) What is the probability that she wins both prizes? ***(2 marks)***

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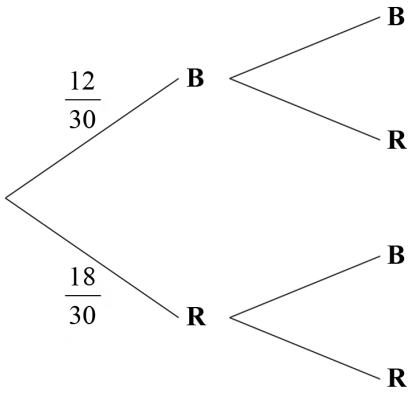
17. Probability, 2UG 2008 HSC 25b

In a drawer there are **30** ribbons. Twelve are blue and eighteen are red.

Two ribbons are selected at random.

(i) Copy and complete the probability tree diagram. ***(1 mark)***



(ii) What is the probability of selecting a pair of ribbons which are the same colour? ***(2 marks)***

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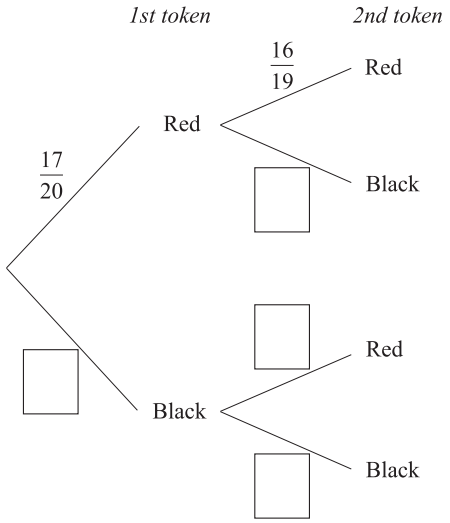
18. Probability, STD2 S2 SM-Bank 01

A game consists of two tokens being drawn at random from a barrel containing 20 tokens. There are 17 red tokens and 3 black tokens. The player keeps the two tokens drawn.

i. Complete the probability tree by writing the missing probabilities in the boxes. ***(2 marks)***



ii. What is the probability that a player draws at least one red token. Give your answer in exact form. ***(2 marks)***

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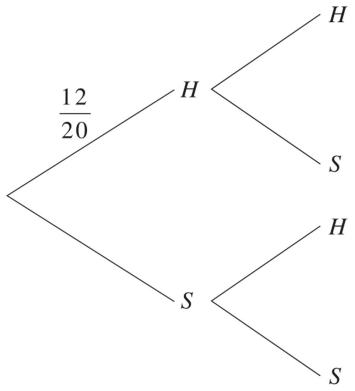
19. Probability, 2UG 2015 HSC 30b

On a tray there are 12 hard-centred chocolates (H) and 8 soft-centred chocolates (S). Two chocolates are selected at random. A partially completed probability tree is shown.



What is the probability of selecting one of each type of chocolate? ***(3 marks)***

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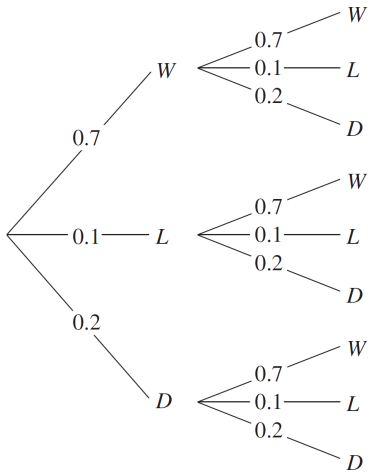
20. Probability, 2UG 2016 HSC 28c

A cricket team is about to play two matches. The probability of the team having a win, a loss or a draw is 0.7, 0.1 and 0.2 respectively in each match. The possible results in the two matches are displayed in the probability tree diagram.



(i) What is the probability of the team having a win and a draw, in any order? ***(2 marks)***

(ii) Paul claims that 1.4 is the probability of the team winning both matches.

Give one reason why this is NOT correct. ***(1 mark)***

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21. Probability, 2UG 2013 HSC 26c

⚠ *Part i: RAP Data - Bottom 2%: School result (4%) was -7% below state average (11%)*

The probability that Michael will score more than 100 points in a game of bowling is $\frac{31}{40}$.

(i) A commentator states that the probability that Michael will score less than 100 points in a game of bowling is $\frac{9}{40}$.

Is the commentator correct? Give a reason for your answer. ***(1 mark)***

(ii) Michael plays two games of bowling. What is the probability that he scores more than 100 points in the first game and then again in the second game? ***(1 mark)***

Worked Solutions

1. Probability, 2UG 2006 HSC 10 MC

$$\begin{aligned}\text{P(Green)} &= \frac{\#\text{ Green chosen}}{\text{Total Selections}} \\ &= \frac{4}{24} \\ &= \frac{1}{6}\end{aligned}$$

$\Rightarrow C$

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2. Probability, 2UG 2007 HSC 10 MC

$$P(\text{hits}) = \frac{2}{7}$$

$$\begin{aligned}P(\text{hits twice}) &= \frac{2}{7} \times \frac{2}{7} \\ &= \frac{4}{49}\end{aligned}$$

$\Rightarrow B$

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3. Probability, 2UG 2014 HSC 19 MC

The tree diagram needs to identify 2 separate events.

1st event - which bag is chosen

2nd event - choosing a red apple from a particular bag

$\Rightarrow A$

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4. Probability, 2UG 2015 HSC 16 MC

$$P(\text{W}) = \frac{7}{10}$$

$$P(\text{WW}) = \frac{7}{10} \times \frac{7}{10}$$

$\Rightarrow D$

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5. Probability, 2UG 2012 HSC 12 MC

$$\begin{aligned}&P\ (\text{at least 1 six}) \\ &\quad = 1\ - P\ (\text{no six}) \times P\ (\text{no six}) \\ &\quad = 1\ - \frac{5}{6} \times \frac{5}{6} \\ &\quad = \frac{11}{36}\end{aligned}$$

$\Rightarrow B$

♦♦♦ *Mean mark 25%*
COMMENT: The term "at least" should flag that calculating the probability of $1 - P(\text{event not happening})$ is likely to be the most efficient way to solve.

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6. Probability, 2UG 2013 HSC 18 MC

$$\text{Total outcomes} = 6 \times 6 = 36$$

$$\text{Outcomes that sum to 6} = (1,5)\ (5,1)\ (2,4)\ (4,2)\ (3,3) = 5$$

$$\therefore\ P(6) = \frac{5}{36}$$

$\Rightarrow D$

♦♦ *Mean mark 35%*

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7. Probability, 2UG 2004 HSC 18 MC

$$\begin{aligned}&\text{P (Only 1 six)} \\ &\quad = P(6, \text{not } 6) + P(\text{not } 6, 6) \\ &\quad = \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \\ &\quad = \frac{10}{36} \\ &\quad = \frac{5}{18}\end{aligned}$$

$\Rightarrow C$

Worked Solutions

8. Probability, 2UG 2010 HSC 20 MC

♦ *Mean mark 48%*

Let $P(\text{Lou successful}) = P(L)$

Let $P(\text{Ali successful}) = P(A)$

$$\begin{aligned} P(\text{only 1 successful}) &= P(L) \times P(\text{not } A) + P(\text{not } L) \times P(A) \\ &= (0.7 \times 0.4) + (0.3 \times 0.6) \\ &= 0.28 + 0.18 \\ &= 0.46 \end{aligned}$$

$\Rightarrow D$

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9. Probability, STD2 2011 HSC 26a

(i) $X = 3 + 2 = 5$

(ii) $P(\text{score} < 4) = \dfrac{6}{12} = \dfrac{1}{2}$

(iii) $P(3) = \dfrac{2}{3}$

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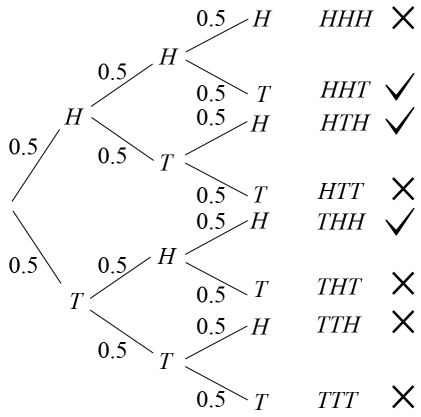
10. Probability, 2UG 2014 HSC 28c



$P(\text{2 heads, 1 tail})$

$$\begin{aligned} &= P(HHT) + P(HTH) + P(THH) \\ &= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \\ &= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \\ &= \frac{3}{8} \end{aligned}$$

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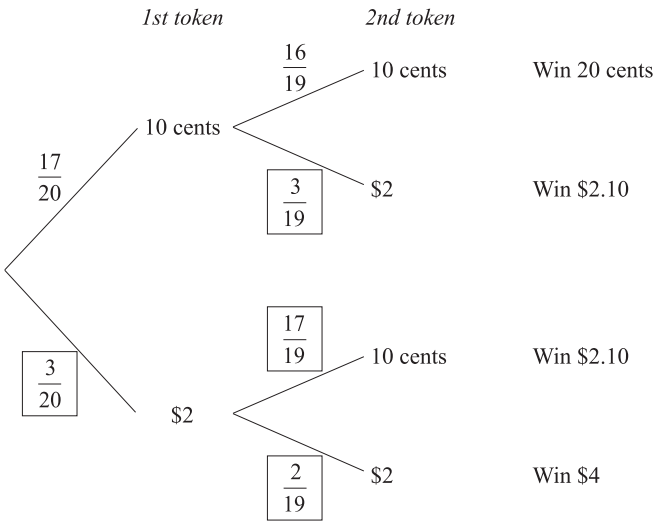
11. Probability, STD2 2018 HSC 30d

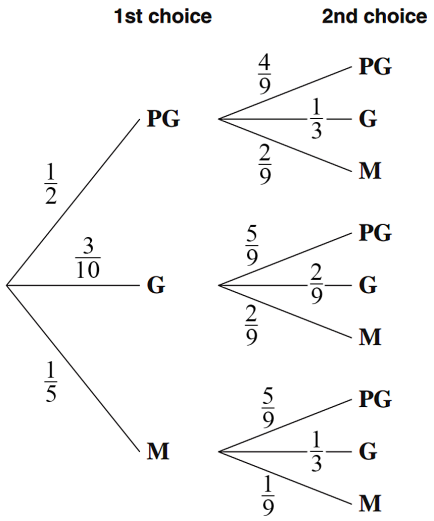


12. Probability, 2UG 2007 HSC 25c

(i) 5 PG, 3 G, 2 M

$$P(\text{M}) = \frac{2}{10} = \frac{1}{5}$$

(ii)



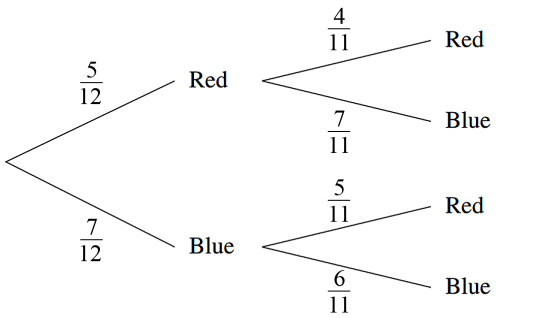
(iii) $P(\text{same rating})$

$$= P(\text{PG, PG}) + P(\text{G, G}) + P(\text{M, M})$$
$$= \left(\frac{1}{2} \times \frac{4}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right) + \left(\frac{1}{5} \times \frac{1}{9}\right)$$
$$= \frac{2}{9} + \frac{1}{15} + \frac{1}{45}$$
$$= \frac{14}{45}$$

13. Probability, 2UG 2005 HSC 23c

(i) $$P(R) = \frac{\#\text{ red ties}}{\#\text{ total ties}}$$
$$= \frac{5}{12}$$

(ii)



(iii) $P(\text{same colour})$
$$= P(\text{RR}) + P(\text{BB})$$
$$= \frac{5}{12} \times \frac{4}{11} + \frac{7}{12} \times \frac{6}{11}$$
$$= \frac{20}{132} + \frac{42}{132}$$
$$= \frac{31}{66}$$

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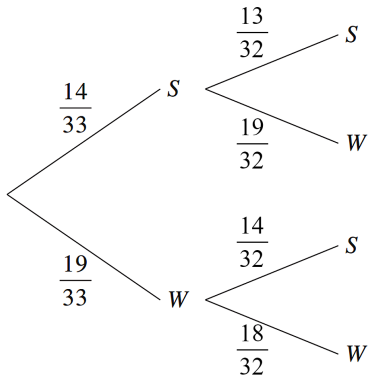
14. Probability, 2UG 2012 HSC 27e

(i)



♦ *Mean mark 43%*

(ii) $P\,(\text{2 silk}) = P(S_1) \times P(S_2)$
$$= \frac{14}{33} \times \frac{13}{32}$$
$$= \frac{91}{528}$$

(iii) $P\,(\text{diff}) = P(S_1, W_2) + P(W_1, S_2)$
$$= \left(\frac{14}{33} \times \frac{19}{32}\right) + \left(\frac{19}{33} \times \frac{14}{32}\right)$$
$$= \frac{532}{1056}$$
$$= \frac{133}{264}$$

♦ *Mean mark 41%*
MARKER'S COMMENT: In better responses, students multiplied along the branches and then added these two results together.

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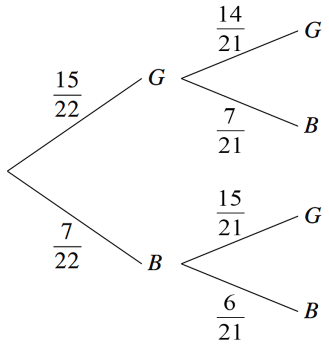
15. Probability, 2UG 2013 HSC 30b

(i)



♦ *Mean mark 40%*

(ii) $P(\text{same gender}) = P(G,G) + P(B,B)$

$$= \left(\frac{15}{22} \times \frac{14}{21}\right) + \left(\frac{7}{22} \times \frac{6}{21}\right)$$

$$= \frac{210}{462} + \frac{42}{462}$$

$$= \frac{252}{462}$$

$$= \frac{6}{11}$$

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16. Probability, 2UG 2006 HSC 25c

(i) $\text{P (wins 1st prize)} = \dfrac{\text{\# tickets bought}}{\text{total tickets}}$

$$= \frac{3}{180}$$

$$= \frac{1}{60}$$

(ii) $\text{P (wins both)} = \text{P (wins 1st)} \times \text{P (wins 2nd)}$

$$= \frac{1}{60} \times \frac{2}{179}$$

$$= \frac{1}{5370}$$

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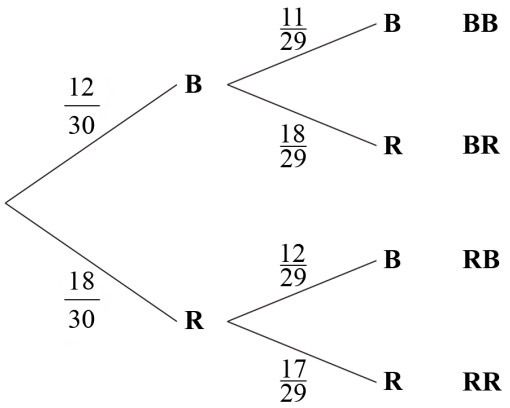
17. Probability, 2UG 2008 HSC 25b

(i)



(ii) P(same colour)

$$= \text{P(BB)} + \text{P(RR)}$$

$$= \frac{12}{30} \times \frac{11}{29} + \frac{18}{30} \times \frac{17}{29}$$

$$= \frac{132}{870} + \frac{306}{870}$$

$$= \frac{73}{145}$$

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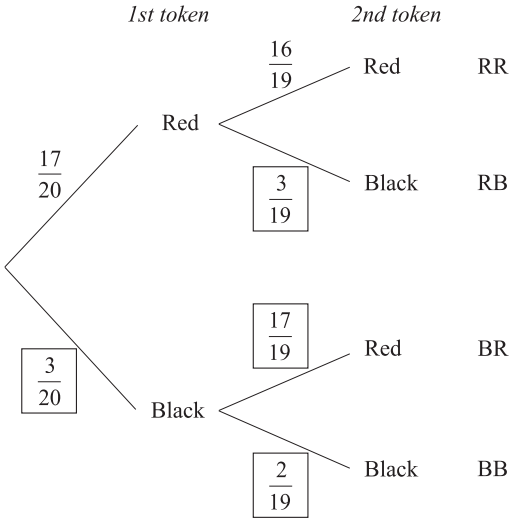
18. Probability, STD2 S2 SM-Bank 01

i.



ii. $P(\text{at least one red})$

$$\begin{aligned} &= 1 - P(BB) \\ &= 1 - \frac{3}{20} \cdot \frac{2}{19} \\ &= \frac{187}{190} \end{aligned}$$

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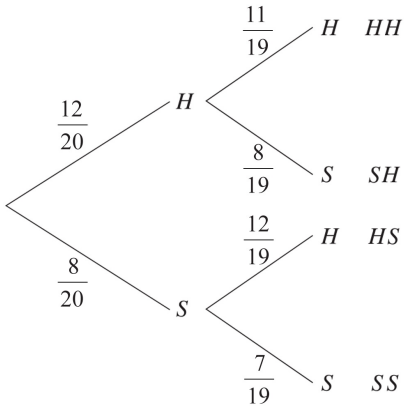
19. Probability, 2UG 2015 HSC 30b



$P(\text{one of each type})$

$$\begin{aligned} &= P(SH) + P(HS) \\ &= \left(\frac{12}{20} \times \frac{8}{19}\right) + \left(\frac{8}{20} \times \frac{12}{19}\right) \\ &= \frac{24}{95} + \frac{24}{95} \\ &= \frac{48}{95} \end{aligned}$$

♦ *Mean mark 45%.*

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20. Probability, 2UG 2016 HSC 28c

(i) $P(W \text{ and } D)$

$$\begin{aligned} &= P(W, D) + P(D, W) \\ &= 0.7 \times 0.2 + 0.2 \times 0.7 \\ &= 0.28 \end{aligned}$$

♦ *Mean mark 45%.*

♦ *Mean mark part (ii) 49%.*

(ii) Probabilities cannot exceed 1.

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21. Probability, 2UG 2013 HSC 26c

(i) **The commentator is incorrect. The correct statement is** $P(\text{score} \leq 100) = \dfrac{9}{40}$

♦♦♦ *Mean mark 11%*

(ii) $P(\text{score} > 100 \text{ in both}) = \dfrac{31}{40} \times \dfrac{31}{40}$

$= \dfrac{961}{1600}$

♦ *Mean mark 34%*